Order Number 9124660

Planning and optimization for logistics management in the food industry

Chung, Hong Kyoon, Ph.D.

The University of Wisconsin - Madison, 1991

U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

A dissertation entitled

PLANNING AND OPTIMIZATION FOR LOGISTICS MANAGEMENT IN THE FOOD INDUSTRY

submitted to the Graduate School of the
University of Wisconsin-Madison
in partial fulfillment of the requirements for the
degree of Doctor of Philosophy

by

Hong Kyoon Chung

Degree to be awarded: December 19___ May 19 91 August 19___

Approved by Dissertation Readers:

John P. Norback
Major Professor

April 5, 1991
Date of Examination

M E Matthews

James G. Morris

John D Wiley
Dean, Graduate School

PLANNING AND OPTIMIZATION FOR LOGISTICS MANAGEMENT IN THE FOOD INDUSTRY

by

HONG KYOON CHUNG

A thesis submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
(Food Science)

at the

UNIVERSITY OF WISCONSIN-MADISON

1991

ACKNOWLEDGEMENTS

I wish to express my warmest appreciation to my advisor, Professor John P. Norback for his very helpful support and guidance throughout my graduate studies and preparation of this dissertation. I also wish to extend my thanks to the members of the examining committee; Professors M. E. Matthews, J. G. Morris, R. D. Hartel, and WM. L. Wendorff for their valuable suggestions and consideration.

Without the support of my families, it would not be possible for me to reach where I am. I am very grateful to my parents who have been a constant source of significant encouragement and care over the years. I especially wish to thank my wife, Kyung Sook, for her love, patience and encouragement. Despite burdens of raising our lovely daughter, Su Jin, and managing home, she has always been supportive so that I could study and complete this dissertation.

I dedicate this dissertation to my parents who have sacrificed their lives for me and my four sisters. They deserve most of whatever I have achieved thus far in life. Finally, I pray for the happiness of my grandmother in the other world, who had wished to visit me in this May.

TABLE OF CONTENTS

Page

Page

Page

LIST OF TABLES

Table		Page

Table Page

LIST OF FIGURES

Figure		Page

Figure Page

LIST OF EXHIBITS

LIST OF NOTATIONS

$[a_{ij}]$	a matrix A
BOM	a Bill Of Materials
B&B	Branch-and-bound methods
CG	Centre of Gravity
C/F	Casein to Fat ratio
DDSS	Distribution Decision Support System
FDM	Fat in the Dry Matter
GP	Gozinto Procedure
GAMS	General Algebraic Modeling System
IP	Integer Programming
JIP	Just-In-Time
LP	Linear Programming
MDS	Matrix Data Structures
MIP	Mixed Integer Programming
MNFS	Moisture in the Non-Fat Substance
MRP	Manufacturing Resource Planning
NLP	NonLinear Programming
NFDM	Non Fat Dry Milk
SNF	Solids-Not-Fat
TSRP	Time-Sensitive Routing Problem

ABSTRACT

PLANNING AND OPTIMIZATION FOR LOGISTICS MANAGEMENT IN THE FOOD INDUSTRY

Hong Kyoon Chung

Under the supervision of Professor John P. Norback at the University of Wisconsin-Madison

Logistics operations in food industry settings are quite different from those in discrete manufacturing industries. Food manufacturers' distinct characteristics relevant to logistics operations led to explore the development of a planning framework suitable for food logistics management and to solve vehicle routing problems in food distribution. Matrix theory and mathematical optimization are proposed as useful bases for developing the framework integrating the flows of materials and information. An example of a hypothetical dairy processor's Cheddar and process cheese plants was used to illustrate and validate the potential use of the framework in food industry logistics management.

Cheese formulations optimized through linear and nonlinear programming were incorporated into bill of materials (BOM) matrices. In a multi-staged, multi-product manufacturing process, gozinto procedure effectively creates the BOM matrix. The BOM matrix flexibly organizes the

direct and indirect relationships of resources to multiple products in various unit measures, and shows how the products compete with one another for common resources in each stage of the manufacturing process. While the BOM matrix may not be suitable for discrete manufacturing industries using a very large number of parts and subassemblies, it provides an appropriate structure to meet the characteristics of food manufacturers. Matrix data structures provide an efficient tool to organize data, obtain desired planning information, evaluate the changes in the information and their impacts on logistics operations, and support management decisions.

Batching is a common practice in the food industries for economic or technological reasons. In a multi-staged batch process manufacturing several products, decisions on how many batches to be produced and whether to produce whole or partial batches with variations in a production target are complex, and have important manufacturing and economic consequences. While product/batch mix decisions under whole batching policies were optimized using mixed integer programming, a penalty approach optimized a product/batch mix when partial batching is allowed. The penalty approach was applied to an example of the production of spaghetti sauce products as an intended guide for building similar models in other industries or for other situations.

Daily delivery of small volumes of perishables to a large number of customers with low margins makes foodservice vehicle routing problems unique. A heuristic approach was used to develop the routing (clustering and insertion) procedures and the allocation of drivers and vehicles. The approach improved the solutions of a previous approach in terms of delivery costs, averaging 5.6% per day of a region, mainly by reduction in the number of routes. Many foodservice customers are located beyond a natural boundary such as a bay. A cluster first - route second approach assigns deliveries to the routes and sequences the deliveries on each route according to a measure of proximity based on straight line distances between deliveries. The measure of proximity without considering the natural boundary often causes erroneous routing schedule in a real distribution situation. A generalized convex combination of delivery points solved the natural boundary routing problem by determining the geographic status of the delivery point. The approaches were incorporated to develop an integrated, interactive computer-based system for routing of foodservice delivery vehicles after being tested with the actual distribution problems.

CHAPTER 1

INTRODUCTION

The food processing industry is one of the largest U.S. industries. The food processing industry is the largest industry group in manufacturing based on the value of the shipments, and the third largest in terms of value added and employment in 1985 (23). The Census statistics classified the food processing industry into nine industries. Table 1.1 shows the value of the shipments for the nine industries from the years 1985 to 1989. In contrast with the erosion in the global market share of such U.S. industries as steel, automotive or consumer electronics, the global position of the U.S. food processors has been very strong primarily due to a large local market size, an abundant supply of high-quality and cheap raw materials, low production costs, and advanced food technologies. Thirteen of the world's 20 largest food processing companies were U.S.-based in 1983 (40). The number of food processing plants, primarily small plants, has been declining, while the size and productivity of the plants have been increasing. The number of food processing plants has fallen sharply by almost 4 percent per year since 1963. While most food processing plants are small, the proportion of large plants are high compared to the other industries (23). More food processors operate multiple plants which specialize in separate products and lines, or the same products in different geographic locations. These industry trends are primarily attributable to the increasing industry consolidation by keener competition, mergers, and acquisition. In general, the size of the plant is influenced by the economies of scale in food manufacturing, supplies of raw materials, and customer demand and location. It is likely that the number of food processors would continue to decrease with an increasing scale of operations coupled with large investments in equipment and technologies.

Characteristics of the Food Processing Industry Logistics

Logistics is defined as the range of activities concerned with the movement of materials through all functional responsibilities from purchasing, to production, to distribution (39). Coupled with an industry characteristic of high volume and low margin, high energy costs, rising inflation, and declining growth rates in productivity, force food processors to improve their logistics operations to maintain a desired level of profitability, and to enhance the product qualities and productivity.

Food processing is generally a capital- and materials-intensive industry. While materials cost accounted for 50.5 percent of the value of 1982 industry shipments, costs of labor and energy are 12.5 and 2.9 percent, respectively(111). Food processors purchase relatively bulky, perishable raw materials and supplies, and transform them into more value-added, storable, palatable food products by applying processing technologies, labor, utilities, and equipment. Then, the food processors distribute the food products to consumers through various channels of distribution. Bulky and perishable raw materials lead to costly physical distribution, which require special material handling techniques, large storage facilities, and fast physical distribution and handling. Besides, quality variation of raw materials requires careful quality control during logistics operations.

Mechanized and automated food processing operations have continuously reduced labor cost and increased productivity. For a reasonable return on significant investments in plants and equipment, facilities should be utilized to a certain level of capacity. It may be, however, difficult to achieve because the production of the raw materials vary yearly and seasonally due to the variations in growing conditions. For instance, milk and egg production is greater in the spring and early summer than in the fall and early winter. A large part of the turkey consumption is observed during the last few months of the year, and the crops like wheat, fruits, vegetables and soybeans are harvested during a relatively short period. Accordingly, food processors often operate at above-capacity rates for a few months of the year and at below-capacity rates for the rest of the year. In this respect, inventory management can be regarded as an area critical to the profitability of the food processors. Even though developments in the technologies of breeding, processing, preservation and transportation have reduced the seasonal variation in production and manufacturing, and improved the shelf-life and convenience of the products, the seasonality of material supplies or product consumption forces many food processors to store a large amount of input materials or finished products. For some products, sales volumes peak at certain times of the year like turkey sales during thanksgiving holidays or ice cream sales in the summer. On the other hand, raw materials, such as fresh vegetables and fruits, may be only available at specific times; thus, canned vegetables and fruits are stored at the end of a canning season. Integrating the logistics operations and managing the product portfolio by manufacturing nonseasonal food products as an extension of the processing season would contribute to reduce the variation in the capacity utilization rate.

To ensure the logistics operations are competently handled and decisions are made in a timely manner and based on accurate information, consistent and flexible information flows between functions must be established. Organizing and manipulating the data, choosing the valuable information accurately and timely, and relating the information to the decision making are a most essential task of the food processors. The data are concerned with the customer demand, materials supplies and prices, materials and product quality, and inventories. The continuing reduction in costs of purchasing and maintaining information systems will allow more food processors to manage the information flows efficiently. The strength in the information management will allow the food processor to manage low-cost logistics operation and put the food processor at a competitive advantage.

The unique material management and the current industry trends of the food processing industry imply that effective logistics management can play a large role in improving the corporate profitability. Each function of the logistics operations has its own objectives and contributions to the corporate profitability. It should be noted that improving efficiencies of individual operation functions such as purchasing, production or distribution are detrimental if the efficiency of the entire system is worse. The key to effective management of logistics operations is integrating information for better decision making regarding the flows of input materials and products systematically and efficiently.

Needs for A Production Planning Framework Fit for the Food Processor

A production manager should continuously decide on the allocation of resources to products, timing of purchasing and product release, and product mix to enhance productivity and profitability. The decision-making is more complicated when the supplies of incoming materials are limited, multi-staged processing is involved, or intermediate products or by-products are used. Without an integrated planning framework, the manager may have to depend on his intuition or experience to make decisions. Decisions stemming from incorrect information would lead to questionable planning and may negatively impact the profitability of the company. For example, excessive inventory ties up capital and increases wastes, whereas insufficient production reduces customer satisfaction and potential profit. By manipulating the production information from an overall perspective and in a timely manner, the production manager can effectively allocate resources, manage material flows and costs, and have a sound basis for the detailed decision-making.

The nature of the production planning is closely related to the type of manufacturing and products. Process industries such as food and chemical industries are mainly concerned with chemical reactions, physical extraction, separation, and/or blending of raw materials. On the other hand, discrete manufacturing industries such as machinery and electrical industries use parts from raw materials and usually combine these parts into subassemblies and products designed to serve specific functional purposes. While discrete manufacturing industries employ production planning and scheduling systems called MRP (Manufacturing Resource Planning) (20), process industries do not fully utilize MRP since MRP is less useful to their manufacturing information needs. Due to distinct characteristics of the industries and products, it is crucial to analyze the unique characteristics of a specific industry and examine their implications for production planning. The characteristics of the food processing industry associated with the production planning and control are summarized as follows:

1. Food processors demands a short lead time due to the perishability or obsolescence of raw materials and products requiring high inventory carrying cost. Since the time lag between production and consumption is generally small, the amount of inventory required is not large. Hence, the food processor's main problem in production planning and control usually lies in managing short lead times. In contrast, the discrete manufacturing industries have long lead times, and the main problem of MRP is trying to manage the time phasing of long lead times (91).
2. The food processors use a small number of resources. Various options of flavors, sizes and packages, however, lead to the product differentiation. For instance, the product using the same ingredient formulation can be packaged in wrappers, glass jars or cans with different sizes. In general, the food products in the same plant use several common ingredients. It is burdensome and inefficient to make separate "make procedures" for a number of products which use many common resources. Intermediate products are often used as a revenue source or are stored for a bottleneck buffer. Some by-products are valuable as a revenue source or the input resources for the finished

or intermediate products. These aspects make it inefficient to follow the practices which the discrete manufacturing industries use to establish a bill of materials (BOM) for each product. Rather, it would be desirable to build an integrated BOM which systematically organizes the recipes of the products with several common resources.

3. The product recipe often requires the resource requirement be measured in units accurate to several decimal places. The precise measurement of material usage is very critical to the assessment of accurate production and inventory costs, and to uniform output products.
4. The measuring units for the same material often vary with the stages of purchasing, processing or storage. According to vendors, raw materials may be purchased as different units such as a 10 pound bag, a 25 pound bag, and so on. Subsequently, these raw materials may be processed using different units such as pounds or ounces. Then, finished products may be sold with numerous package forms and sizes. To accurately control the production and inventory, the conversion relations between several forms of units must be defined and managed.

5. Changes in availability, quality and prices of raw materials, regulations on materials and processing methods, or consumers' food consumption trend may make it necessary to change product recipes. High volume and narrow profit margin of food manufacturing emphasize the timely and efficient control of these changes to assess the impact on the product profitability.
6. Product yield may vary with the quality attributes of ingredients, the use of substitute or processing conditions. It is very important to identify the source of changes in yield and reflect it for proper production planning and quality control.
7. Consideration should be given to the seasonality of raw materials supplies, prices, production and consumption.
8. Batch production is still common in the food processing industry because of technical and economic reasons. The variation in production targets with a discrete production process for multiple-staged and multiple products implicates difficulties in planning and decision-making for producing whole or partial batches. Batching decisions directly affect the total volume of finished products, total resource requirements, and unit costs of products.

A relatively small scale of individual food processors has been a factor limiting the development of the production planning framework meeting their needs. Rather, the food processors have been more interested in computerizing parts of the production system. However, industry consolidation by merger and acquisitions forces the food processors to make more tough management decisions to survive. In view of the general characteristics of the food manufacturing and control, matrix data structures and mathematical optimization provide effective bases for a production planning framework which addresses these typical characteristics and problems.

Matrix Theory Application to Production Planning

Matrix theory is an effective base for developing a food production planning framework by providing an analytical structure to organize, manipulate, and produce the production planning information. A matrix is a rectangular array of numbers (6). The numbers in the array are called the entries. The size (dimension) of a matrix is determined by the number of rows and columns in the matrix. Unless otherwise explicitly stated, matrices are denoted by bold-faced capital letters, vectors by lower case letters with underlines, and matrix entries occur in a rectangular box. The size of the matrix is denoted as m x n or m by n, where m and n are the number of rows and columns, respectively. A shorthand notation for identifying a matrix A is $[a_{ij}]$. The element in row i and column j is a_{ij} for $i \in I$ and $j \in J$. Basic matrix definitions and matrix operations that are applied to the production planning framework are: identity matrix, null matrix, matrix equality, addition, subtraction, multiplication, inverse, transpose, and submatrix.

Matrix theory was used to derive standard production costs (118), to organize multiproduct production information (58), and to develop an optimal production schedule (31). Matrix Data Structures (MDS) and Gozinto procedure (GP) are applications of the matrix theory, and provide useful foundations for the production planning framework. These applications offer analytical means of evaluating the changes in information and presenting their impacts on production planning and control including production costs and product requirement.

Matrix Data Structures (MDS)

Matrix Data Structures or MDS provides an efficient means to organize a variety of data, obtain desired information by manipulating matrix operations. The advantages of MDS are its flexibility, organizational capability, consistency, and computational speed. MDS was employed to organize and manipulate the data on food formulation, resources, nutrition and safety, resource unit costs and production plans (80, 81). MDS was useful to organize and manipulate unit conversion among purchasing, issue and use unit of ingredients in foodservice operations (14). By using MDS to labor requirement information in foodservice operations, computers provided decision support information (95).

Gozinto Procedure (GP)

MDS was successfully applied to a single-staged food manufacturing process (91), whereas the requirement of resource matrices equal to the number of stages leads to the inefficiencies in applying MDS to the multi-staged manufacturing processes. Gozinto Procedure or GP is a systematic procedure based on matrix theory (116, 117), and serves the planning functions of a multi-staged, multi-product manufacturing system (78). The GP is a way to organize production information into a lower triangular invertible matrix. The resulting matrix of GP provides the resource requirement information for every stage of the manufacturing process for multiple products. While the BOM matrix may not be most suitable for discrete manufacturing industries using a very large number of parts and subassemblies, it provides an appropriate structure to meet the characteristics of the food manufacture. As an extension to GP, a BOM matrix with fewer dimensions is created by removing the columns representing the ingredients except intermediate products. The columns representing the intermediate products support the understanding of product recipe structures including the level of ingredients, and the differentiation of direct or indirect ingredients. As a result, the BOM matrix will be of a smaller size than the resulting matrix of the original GP, which enables computer users to more quickly store, retrieve, and manage information.

A Bill of Materials(BOM) Matrix and Its Application to MDS

Every manufactured product is associated with a bill of materials (BOM). The BOM is a record containing the information to identify each input resource and its quantity used to produce a certain unit of the product. Without the BOM, it would be difficult for the people in purchasing and production to know what and how much materials and supplies they should buy and bring them to the manufacturing so as to meet the production plan. To build a BOM, the relationships among the product and input resources must be explored. It is important to note that the product structure relationship should be established based on a level-by-level hierarchy in which a high-level item is the parent of lower level items. There are numerous ways to describe the BOM. One way is a product structure tree, or a BOM tree. In the BOM tree, level 0 is the highest level which indicates the finished product or the end item. As the level number becomes higher, the product structure becomes more complicated.

The BOM tree for a strawberry frozen dessert formula (SD) is illustrated in Figure 1.1. Strawberry frozen dessert formula (SD) is made of two units of Strawberry dessert base (SB), one unit of nuts (NUT), and one unit of strawberry flavor (SF). SB, in turn, is made of one unit of SF and one unit of ice milk mix (IM). SB is an ingredient of SD as well as a parent of SF and IM. Thus, SB is an intermediate product, which is defined as an item having at least one parent and at least one resource or child.

Discrete manufacturing, and food and other process industries are often differentiated by their BOM tree structures (28, 108, 109). Discrete manufacturing industries use thousands or hundreds of parts and subassemblies to make products. In contrast, food processors use much smaller number of raw materials to manufacture food products, and the products usually use several common raw materials or similar formulas. Combining a few raw materials in different proportions, quantities, and/or processing conditions can result in large numbers of finished products. In addition, various options of flavors, sizes and packages lead to several different finished products. For instance, frozen desserts with different flavors can be packaged in cups, cones, waffles, or cartons with different sizes. While the BOM tree of the discrete manufacturing industries has a pyramid form, that in the food and other process industries generally has a quite different form, often called an inverted BOM (28, 108).

Figure 1.2 illustrates the BOM tree for strawberry frozen dessert products differentiated by the size of the cone. Strawberry frozen dessert in a large cone (SD-LCN) is made of two units of strawberry frozen dessert formula (SD) and one unit of large cone (CN-LG). Strawberry frozen dessert in a small cone (SD-SCN) is made of one unit of strawberry frozen dessert formula (SD) and one unit of small cone (CN-SM). This BOM tree can be extended as more options of packaging materials or sizes are added. A good example of the inverted BOM can be found in meat and poultry processing, where a major ingredient like a turkey is processed into several parts with optional ingredients.

The BOM tree structure is useful to understand the structural complexity of the product recipe or formulation, but it is not convenient for recording, storing, or manipulating in the computer data base. The BOM tree will be more complicated as more similar products and ingredient options are added. Thus, manufacturers use a BOM file as a record form. There are two major types of BOM files: indented BOM file and summarized BOM file (47). The indented BOM file lists the materials based on their levels by indenting the materials which go into higher levels. The summarized BOM file enumerates the product and materials with total quantity requirement per unit of the end item.

Figure 1.3 shows these two different BOM file forms for SD. The indented BOM file shows structural relationships between the product and materials, while the summarized BOM file allows a shorter listing and less computer storage. The summarized BOM file may be desirable when the same material is used repeatedly throughout a product structure (30). However, both BOM files show only one BOM for one end item. In the food industry, it is burdensome and inefficient to make separate BOM files for a number of finished or intermediate products which would use slightly different resources with one another. A useful alternative to the BOM tree and file for the food processor is the BOM matrix. While the BOM matrix may not be most suitable for discrete manufacturing industries using a large number of parts and subassemblies, it provides an appropriate structure to meet the characteristics of the food manufacture. The BOM matrix offers a convenient way to define the relationships of resources to products in the food industry, to systematically integrate multiple product formulas, and to show how the products would compete with one another for common resources. The BOM matrix can define the relationships of resources to products in a variety of different unit measures. For instance, batch production is still a common manufacturing practice in the process industries (7). When the batch production is involved in manufacturing process, the production department generally uses a batch formula, while marketing department uses a packaging unit basis to forecast the demand or distribute the product. The BOM matrix can flexibly organize the different formula bases.

Food processors pay a lot of attention to managing and controlling the flows of intermediate products or by-products. Intermediate products are stocked as a buffer for bottleneck situation or used as a means for preservation instead of more perishable raw materials (110). Some intermediate products are sold or processed further to manufacture more value-added products. While some by-products incur disposal costs, others are sold as a revenue source or used as a resource for the finished or intermediate product manufactured in the same plant or other plants. For example, whey cream is a by-product from the production of natural cheese and is often re-incorporated into other products such as processed cheese. In the multi-staged process, it is important to know what and how much ingredients are used to make a unit of intermediate product as well as to make a unit of finished product, and how much by-product results from a unit of finished or intermediate product. The BOM matrix meets such needs of the food processors. In the BOM tree for SD-LCN and SD-SCN, SF(1) is used to indicate that one unit of SF is required to make one unit of SD and two units of SB. The BOM tree does not show how many units of SF are required to make one unit of SD-LCN or SD-SCN. The situation is the same in the indented BOM file. In the BOM tree or the indented BOM file, input resource requirement for one unit of end item is determined by multiplying the unit values for each resources. To know SF requirement for producing one unit of SD-LCN and SD-SCN, the following computation is needed: (1*2*1 + 1*2*2*1) + (1*1*1 + 1*1*2*1) = 9 units of SF. Such a computation becomes more cumbersome as more products, resources, or relationship levels are involved. While the summarized BOM file shows the resource requirement for one unit of end item, it does not provide any structural information about the product recipe. There is no way to tell what materials go together to make the end item. On the other hand, BOM matrix shows not only structural information, but also how many units of SF are used to make one unit of SD-LCN, SD-SCN as well as SB and SD. Figure 1.4 shows the integrated BOM matrix **B** for SD-LCN, SD-SCN, and their intermediate products, SD and SB.

Once a BOM matrix is established, additional supportive information can be derived by manipulating the MDS. Several uses of the MDS for dealing with food manufacturing processes propose that the BOM matrix applied to the MDS provides advantages over using MDS in isolation. The procedures together can more efficiently manage the planning function of multiple production stages associated with many products and resources. For example, it offers a good means to quickly compute the resource requirements for each planning period. To determine the resource requirement for a production plan of SD-SCN, for instance, a planning matrix **P** is created. The size of the matrix **P** is determined by columns equal to the number of finished and intermediate products, and rows equal to the total number of production periods or plans. The entries p_{ij} represent the number of units of product j required in plan i. To derive resource requirement for the first three month production plan for SD-LCN and SD-SCN, the product of the matrices **B** and **P** is taken in Figure 1.5. The resulting matrix **L** organizes the total resource requirements for the production plan.

Unit product costs, per stage processing costs, intermediate product costs, and production costs for each period can be also easily obtained. To derive unit product cost, a vector $\underline{r}$ organizing the resource unit costs is created and multiplied by the BOM matrix **B** as shown below.

$$\underline{r} = [\ 0 \quad 0 \quad .05 \quad .03 \quad 0 \quad 0 \quad .04 \quad .02 \quad .10\ ]^t$$

	SD-LCN	SD-SCN	SD	SB	
$\underline{r}^t\mathbf{B}$ = [	.65	.33	.30	.12	] = $\underline{u}^t$

The vector $\underline{r}$ has the entries equal to the number of rows in the BOM matrix **B**. It is important to note that the costs of intermediate products SD and SB should not be recorded in the vector $\underline{r}$ because they will be computed by their resource costs. The resulting vector $\underline{u}$ organizes the unit cost for finished and intermediate products. To derive total ingredient cost for each month, the product of the cost vector $\underline{r}$ and the matrix **L** is taken:

$$\underline{r}^t\mathbf{L} = \begin{matrix} \text{Jan} & \text{Feb} & \text{Mar} \\ [\ 114.5 & 261.3 & 291.0\ ] \end{matrix}$$

To obtain total ingredient cost for the product at each month, the unit finished product cost is multiplied by the matrix **P** in Figure 1.6. By summing up the row entries for each column, total ingredient cost can be also obtained. Even though the resulting matrices show ingredient cost, total direct production costs can be obtained by adding unit labor and utility cost for each product to the matrices. Additional quantified planning data can be incorporated into the matrices for obtaining useful production planning information. By manipulating the MDS with the inventory matrix, inventory information can be managed. Product unit profit contributions can be also computed by manipulating product unit price vector and resource unit cost vector. MDS was useful to organize and manipulate unit conversion among purchasing, issue and use unit of ingredients (14). The BOM matrix and its application to MDS provide a good base to build the production planning framework well fitted to the needs of the food processors.

Several departments in the company may use different BOM structures or contents according to their own purposes. Marketing and distribution departments use the units such as cases or boxes for forecasting or distribution purposes, while production department would be more familiar with the production volume or the number of batches. The accounting department often has its own BOM for cost accounting purpose. It is unlikely that the departments have the same BOM. They may have different structures, definitions, descriptions, or unit of measures. This situation can generate confusion and errors in production, marketing and financial planning and their implementations, which would lead to the conflicts about the information flow, materials and products, and a lack of information credibility among the departments. If the company use a common BOM structure with an integrated database for the definitions of BOM components, and unit conversion matrices between different units of the departments, it would significantly improve productivity, and reduce operating costs and time. To create and maintain the accurate, consolidated BOM matrix, however, the coordination among the departments is essential.

The matrix offers a good structure for computer programming and manipulation. The computational and sparse structure of the matrices makes the computer manipulation more efficient. Computerized MDS offers the practical microcomputer-based planning (91). Gozinto procedure can be also programmed without difficulty due to the sparse and lower triangular structure of the matrices. This fact implies a computerized production planning framework based on BOM matrix application to MDS. The use of computers for BOM application to MDS suggests that the food processor can extract timely information on production planning and control. It also provides the flexibility to make changes and evaluate them whenever needed. Once the BOM matrix is established, each department can retrieve its needed data by selecting a specific subdimension of the matrix, and manipulating it with MDS to obtain the desired information. When the food processor manufactures the product families which use few common materials, it would be more efficient to have separate BOM matrix for each product family.

The commonality of the BOM matrix among the user departments can yield benefits to each department and therefore the company. Timely and precise monitoring for the resources can save purchasing costs for less obsolescence and large quantity purchasing. Better quality control can be achieved because attentions are paid to monitoring the resources and processes in an organizational way. This will also benefit production costs. With better resource allocation and inventory control, a higher utilization rate of the equipment can be achieved by controlling the equipment to flexibly produce a variety of products. Accounting departments can quickly and accurately evaluate the impact of the changes in resource unit requirement or cost on the production cost.

Mathematical Optimization

A mathematical model is one that provides a concise framework for analyzing a decision problem in a systematic manner. In this respect, the objective and decision variables of the system and the constraints on the system are the basic components that are essential for constructing a model (22). The mathematical model is a mathematical representation of the system which needs optimization. Optimization requires choosing the best value among all possible alternatives in a given situation, according to a performance measure for objectives such as maximizing profit or minimizing cost. The general mathematical model can be written in the following form:

OPTIMIZE $Z = f(\underline{x})$: *Objective function*

SUBJECT TO

$$\left.\begin{array}{ll} g_i(\underline{x}) \leq b & i = 1, 2, ---, m \\ h_j(\underline{x}) = c, & j = 1, 2, ---, l \\ \underline{x} \geq \underline{0} & \end{array}\right] : \textit{Constraints}$$

where $\underline{x}$ is a vector of n *variables* $x_1, x_2, ---, x_n$.

It is important to note that the optimal solution of the model is the best only relative to the model, not the decision problem because of possible interaction of the uncontrollable variables or quantifying errors of some variables regarding quality attributes. It is often difficult to describe and optimize processing phenomena and production environments mathematically and economically. In the production of bulk cheese cultures, for example, chemical and biological reactions occur over a controlled period of time. The rate of reactions depend on various processing conditions and some interactions between ingredients which may accelerate or retard the reactions. Many interaction effects are not linear. The nonlinearity can be also identified in the profit contribution of the ingredient whose cost is likely to depend on the purchasing amount. However, the problem can be described much more concisely and comprehensively by the mathematical model (51). Also, the mathematical model facilitates dealing with the problem in its entirety and considering all the interrelationships simultaneously. Furthermore, the model facilitates the use of powerful mathematical techniques and computers to analyze the problem.

In addition to mathematical models, simulation and heuristic models are used. Simulation models imitate the behavior of the system and measure the performance as statistical observations. Simulations do not need explicit mathematical function so it can simulate complex systems which can not be modeled or solved mathematically. A heuristic procedure is used when the mathematical model is too complex to allow an exact solution. Heuristic methods rely on intuitive or empirical rules which determine an improved solution relative to the current one but do not guarantee an optimal solution. Once a model is available, it is necessary to have a suitable algorithm and good computer codes to solve the optimization problem.

In the food industry, the decision problems for optimization include product formulation, production planning, production scheduling, plant design, equipment design, process optimization, optimization of operating conditions, and so on (13, 18, 27, 59, 97, 98). In general, constraints deal with plant capacity, resource availability, equipment and storage, yield, quality attributes, and legal requirement.

Mathematical models are used in many operations research methods. Among them, linear programming model is the most popular application in the food industry. As mentioned earlier, some decision problems cannot be described linearly. In these cases, nonlinear programming or integer programming is often used (10, 36, 42). In this thesis, an optimization approach is used to find the formulation of Cheddar cheese and product/batch mix of Cheddar cheese, process cheese products, and by-products. In optimizing product mix, integer decision variables are used to represent the number of whole batches.

Integer Programming Approach

Any decision problem in which the decision variables must assume discrete values may be classified as an integer programming (IP) optimization problem. In general, an IP may be constrained or unconstrained, and the functions representing the objective and constraints may be linear or nonlinear. In other words, IP methods seek the determination of the optimum point among all the discrete points included in the continuous feasible solution space.

The linear IP problem may be stated in maximization form as follows:

$$\text{Maximize } f(\underline{x}) = \underline{cx}$$

Subject to

$$\mathbf{A}\underline{x} \le \underline{b}$$

$$\mathbf{B}\underline{x} \le \underline{d}$$

$$\underline{x} \text{ integer.}$$

Ignoring the integrality condition, this IP problem becomes linear program and the solution space is convex.

IP problems are classified into pure IP and mixed IP (MIP). While all decision variables are restricted to integer values in the pure IP, some of the variables are continuous in the MIP.

IP problems often arise in the food industry because some or all of the decision variables must be restricted to integer values. For example, integer decision variables makes more sense when the number of package products, whole batches, employees, and machines are assigned to activities in integer values.

IP problems are generally much more difficult to solve than linear programming (LP) or nonlinear programming (NLP) problems without the integrality restriction. Even though numerous algorithms have been developed and continuously improved for solving IP problems, the algorithms are not uniformly performed and are much less efficient than those for LP or NLP.

Computer codes for IP are available in many mathematical programming software packages. However, the solution time for IP problem is often unpredictable and finding a solution is sometimes impossible, particularly when the number of integer variables is large. One of the major difficulties in IP computation is the effect of round-off error that results from the inevitable use of the computer because the digital computer handles computations in floating point arithmetics only (104). As the iterative computations continue, the effect of accumulated roundoff error increases. The computational difficulty has forced some users to solve the problem as a continuous model by simply applying simplex algorithm as a LP-relaxation, and then round the continuous optimal solution to a feasible integer solution. For instance, if the continuous optimal solution indicates that the number of batches required is 7.2, this number can be rounded to 7. But there is no guarantee that the rounded solution will always satisfy constraints as in the case that several types of batches or products, and some constraints for demand and supplies or equality constraints are involved. Thus, every integer problem cannot be handled in this way because it may be difficult or impossible to find a feasible integer solution, or the solution found may be far from the optimal solution.

In general, the IP methods are classified into cutting methods and search methods. Cutting methods are developed for the integer linear problem but are not reliable, regardless of the size of the IP problem (103). Search methods include zero-one implicit enumeration, and branch-and-bound methods. Branch-and-bound (B&B) methods are most reliable among the methods, and most commercial codes are written based on these methods. B&B method first solves the pure or mixed IP problem as a continuous model because it is simpler to deal with a linear problem (62). If the optimal continuous solution is all integer, then it is also optimum for the IP because the solution space of the IP is a subset of the continuous space. Otherwise, the B&B method resorts to an intelligent search of all possible solution points by branching and bounding. Branching process deletes parts of the continuous space that do not include feasible integer points by enforcing necessary integrality conditions, while bounding locates optimum integer solution by discarding inferior subproblem. The details of these methods are well explained in numerous articles and books (36, 42, 61, 62, 94, 103).

FOOD DISTRIBUTION

Food distribution management is applied to the outgoing product flow from the company to customers through a distribution system. The U.S. food distribution markets reached $78 billion in 1985 (29). The goal of the food distribution is to deliver food products with the desired qualities in the right quantity at the right time to customers efficiently and reliably. The distribution system includes storage and transportation network. Distribution management is a critical factor for attaining the effective and efficient marketing of the product. It should be however noted that improving efficiencies of individual operation functions such as purchasing, production or distribution are useless if the efficiency of the individual function disrupts systemwide optimization. To satisfy customer needs and keep distribution costs competitive, distribution managers must understand and manage not only the physical flow of food products, but also the information flow for purchasing, production, and inventories.

In an economy characterized by high energy costs, rising inflation, potential materials and energy shortages, and declining growth rates in productivity, maintaining desirable levels of corporate profitability is becoming increasingly difficult. The distribution function offers a great potential for profit improvement. In many industries, distribution costs exceed 25 percent of each sales dollar at the manufacturing level (101). Distribution costs are particularly enormous in the food industry. For instance, the distribution costs of the soft drink sector comprised about 32% of the cost of sales (45).

Distribution costs involve both visible and hidden costs. Visible distribution costs include transportation costs, order processing and information costs, inventory carrying costs and warehousing costs. Hidden distribution costs are profit opportunities lost due to the costs of canceled orders and customer dissatisfaction associated with stockout and failure to deliver the product on time. Hidden opportunity costs also occurred when the company does not utilize the corporate distribution assets. Distribution managers likely disregard the hidden costs due to the hidden characteristics of the costs.

Building an effective, reliable distribution system requires the development of a desired customer service level, the selection of transportation modes, the determination of the optimal number and location of plants, warehousing facilities or distribution centers, the design of an efficient order processing and information system, and the establishment of vehicle routing and scheduling, and inventory control systems.

Determination of the customer service level

An immoderate on-time delivery schedule may increase customer service but could increase transportation costs and inventory carrying costs. On the other hand, the effort to lower transportation costs through tight delivery schedule or increase in order lead time may reduce customer service level. It is therefore critical to achieve the balance between distribution cost and the level of customer service. To attain the balance, distribution manager must determine the reasonable level of customer service and aim at minimizing the total costs of distribution at the given level of customer service.

The customer service level must be set according to customer needs. In developing a desired level of customer service, it is critical to determine important elements of customer service, which can be obtained through customer service survey. The importance of service elements vary from industry to industry, and even from company to company within an industry. In the food distribution, customers frequently need deliveries of small volume of food products within one or two days, mainly for inventory control of fresh products. The critical success factors for customer satisfaction in the food distribution may include timely and reliable delivery of products with the desired qualities, efficient and convenient order processing system, short order lead time, appealing product packaging, and quick settlement of customer complaints.

Once the critical service elements are determined, a desired level of customer service must be developed and implemented through corrective action to reduce the discrepancies between actual and desired performance. For example, a specific service level such as 95 percent for cases shipped over cases ordered during a 2 day lead time may be established. Different customer service levels can be established according to the products and customers. For instance, highly perishable, popular, or profitable products would have higher service levels as well as higher inventories. The order lead time may be different depending on the customer location or demand.

Selection of transportation modes

Transportation modes account for a major proportion of the food distribution costs. Perishability of food products largely determines the transportation mode. Perishable products require an expensive preservation system such as a refrigerated system during warehousing and transportation. Such transportation qualifications and its maintenance demand a substantial amount of corporate asset and costs, however.

There are a variety of transportation options for distributing food products. The four basic modes of transportation are truck, rail, water, and air. In addition, a variety of combinations are available such as rail-motor (piggyback), motor-water, and motor-air. The most popular mode in the food distribution is trucking, because it provides door-to-door service and flexibility in scheduling. But trucking may not be efficient for a long distance distribution because of restrictions on food safety and high costs. Rail movement is considered less desirable by food companies because of its inability to meet time-sensitive delivery and intermodal transportation needs. Many food companies turned from rail to trucking in order to meet narrow service time windows created by just-in-time distribution networks (74). Rail transport is still popularly used as a major long-haul mover for low-value bulk food products, however. For food products that are not time-sensitive and are traveling 500 miles or more, rail is a widely used option (26). Developments in insulation technology enabled food companies to transport perishables 2,000 miles or more on rail by keeping track of perishables using phone-activated tracing system (41).

In choosing a transportation mode, distribution managers must consider product characteristics, cost, speed, dependability, and possibility of loss and obsolescence associated with the modes available to them. An emerging trend is the declining importance of cost as the criteria for purchasing transportation equipment (74). A total logistics approach toward distribution places more importance on the quality of service and equipment, with cost being the third.

Determination of the optimal number and location of plants and warehousing facilities

The number and location of the food distribution center are influenced by geographic dispersion of customers, product characteristics (bulkiness, perishability, seasonality, substitutability, market concentration), customer service level, transportation costs, inventory carrying costs, and costs of operating distribution centers. The number of distribution centers increases when customers are geographically dispersed, products are perishable, customer service level increases, reducing transportation costs is a main goal of the distribution management, inventory carrying costs are relatively low, and operating costs are low.

The location of plants and distribution centers have a significant impact on the competitive position of the company. The location of the plant and distribution center generally depends on the costs of transporting raw materials to the plant and those of shipping the finished products to consumers. If the raw material is even bulkier than the finished product, the transportation of the raw material is expensive, and the unit value of the finished product is higher, the plant and distribution center will be likely to be located near the source of the raw material and consumers, respectively. This is the case for flour milling, meat packing, and cheese manufacture. On the other hand, if the finished product is perishable and the transportation of the product is expensive, the plant will tend to be located near to consumers, as with baking, milk and ice cream plants. The truck shipment of orange juice concentrate from Florida to northern markets for packaging is another example of moving a product closer to the point of consumption to overcome the value and bulk restrictions. The food processors will attempt to minimize the total costs of raw material and finished product transportation. The readers who are interested in warehousing facility location models and algorithms are referred to Love, Morris and Wesolowsky (71).

Design of an order processing system

The customer order cycle includes total time consumed by order preparation and transmittal, order receipt, order entry, order processing, warehouse picking and packing, order transportation, and delivery and unloading at the customer's dock. Thus, the length of the customer's order cycle is determined not only by the speed of the movement of the products, but also by the speed and efficiency of the information flows between a supplier and customers.

Owing to high storage and labor costs, and rapid labor turnover, food manufacturers and distributors increasingly use computer-based automatic order selection system. An on-line computerized ordering system can achieve faster order cycles, and reduce order lead time, storage costs, ordering error, and stockouts. A food company may use a voice-response order entry system, which allows telephones to act as terminals to the company's host minicomputer for faster and more precise order entry (2).

Establishment of vehicle routing and scheduling, and inventory control systems

Advancements in distribution technologies present opportunities for cost savings. These technologies include computer-based order processing, inventory control, and vehicle routing and scheduling systems. The computer-based inventory system coupled with the highly mechanized material handling system tracks all materials in and out of storage, provides materials with inventory updates and location in real time and reduces breakage of finished products.

Vehicle routing and scheduling are one of the most commonly occurring problems in the distribution management. Traditionally, human dispatchers address these problems, but there has been a significant development in computer-aided systems to assist human dispatchers. Computerized vehicle routing and scheduling systems save the organization a considerable amount of operating costs and help improve the operations of the organization through improved vehicle utilization and a high level of customer satisfaction by reliable and on-time delivery (12, 19, 34, 38, 45, 54). The man-machine interactive computerized system can provide flexible routing and scheduling. For example, the human scheduler may relax some soft transportation constraints which the computer may not allow to violate. Heuristics have been popularly used to attempt to overcome the difficulties of complex large problems or specific industry problems by using the understanding of the specific problems (21, 37, 43, 93, 99, 105, 120). Chapter 4 described the application of a man-machine interactive heuristic to the foodservice distributor.

The goal of the corporate distribution strategy can change or be outdated. Changes in the following factors may indicate a need for strategy revisions: demand, competition, geographic distribution of customers, customer service level, processing and transportation techniques, product characteristics, proportion of distribution costs in sales, and pricing policy. The food industry has a high percentage of distribution costs in sales. Hence, even small changes in fuel prices, driver pay, and interest rates can make distribution strategy modification worthwhile. Food distribution costs are also sensitive to product characteristics such as weight, volume, and shelf life. Instead of using common carriers, more food and foodservice companies develop and use their own transportation systems to enhance serviceability and profitability. Adoption of cost-efficient technologies and flexible distribution management in response to the changes in market conditions is critical to maintain or gain an edge over its competitors in the high volume and low margin food distribution industry.

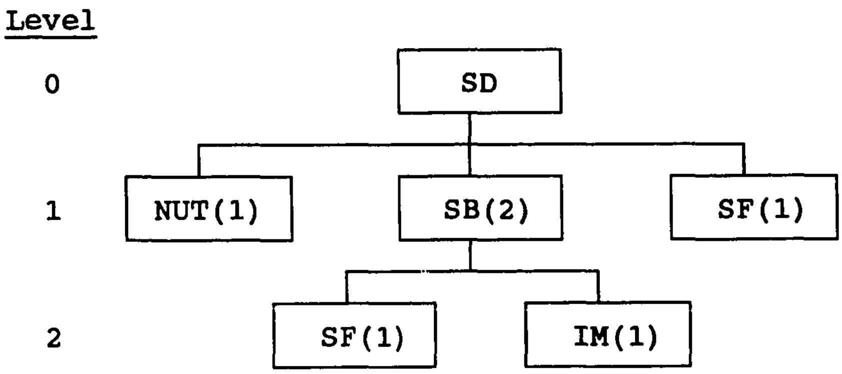
Table 1.1. Census statistics for the food processing industry

Industry	Value of shipments ($ million)[a] 1985	1986	1987	1988	1989
Meat products (201)	66,075	67,898	69,346	74,616	78,813
Dairy products (202)	41,639	42,550	43,866	39,685	42,613
Preserved fruits and vegetables (203)	36,186	36,348	37,816	36,896	40,442
Grain mill products (204)	35,078	35,754	37,283	35,714	39,043
Bakery products(205)	21,064	22,226	29,979	22,813	25,284
Sugar and Confections(206)	18,161	18,695	18,059	19,426	20,420
Fats and oils (207)	20,977	21,918	16,298	16,707	17,183
Beverages(208)	42,713	44,123	47,758	47,482	50,040
Miscellaneous foods(209)	25,434	27,169	28,588	30,215	32,064
Total	307,324	316,681	320,991	323,554	345,903

( ): SIC (Standard Industrial Classification) code of industries

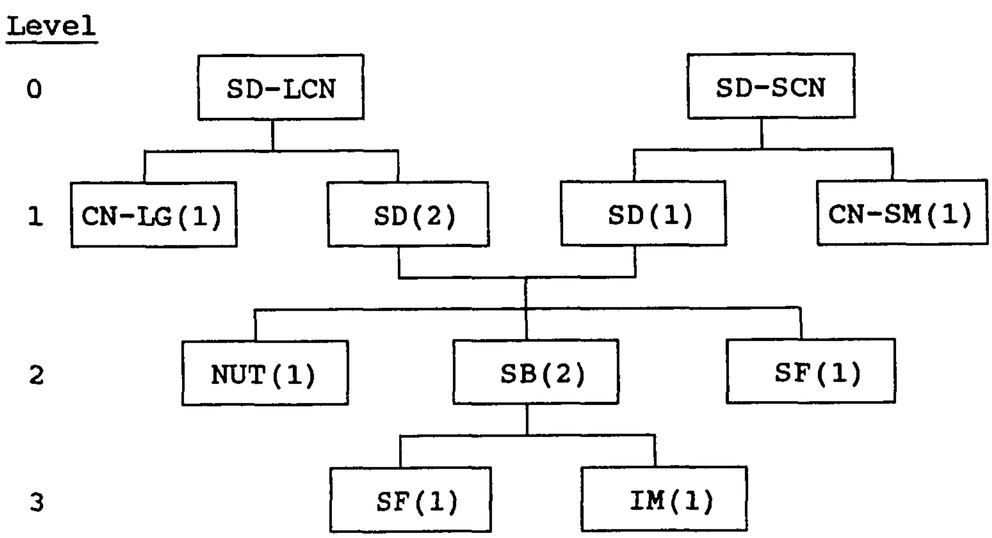
[a] Value of Shipments: received or receivable net selling values, f.o.b. plant of all products shipped as well as all miscellaneous receipts.

Source: U.S. Department of Commerce, Bureau of Economic Analysis, and International Trade Administration, Washington, DC 20233.



SD: strawberry frozen dessert formula
SB: strawberry dessert base
NUT: nuts
SF: strawberry flavor
IM: ice milk mix

Figure 1.1. BOM tree for Product SD



SD-LCN: strawberry frozen dessert in a large cone
CN-LG: large cone
SD-SCN: strawberry frozen dessert in a small cone
CN-SM: small cone

Figure 1.2. BOM tree for products SD-LCN and SD-SCN

Indented BOM file for SD

SD: Strawberry Dessert Formula

Item	Unit of Measure	Quantity
SB	unit	2
SF	unit	1
IM	unit	1
NUT	unit	1
SF	unit	1

Summarized BOM file for SD

SD: Strawberry Dessert Formula

Item	Unit of Measure	Quantity
SB	unit	2
NUT	unit	1
SF	unit	3
IM	unit	2

Figure 1.3. Indented BOM file and summarized BOM file for SD

	SD-LCN	SD-SCN	SD	SB	
B =	1	0	0	0	SD-LCN
	0	1	0	0	SD-SCN
	1	0	0	0	CN-LG
	0	1	0	0	CN-SM
	2	1	1	0	SD
	4	2	2	0	SB
	2	1	1	0	NUT
	6	3	3	1	SF
	4	2	2	1	IM

Figure 1.4. BOM matrix **B** for SD-LCN, SD-SCN, SD, and SB

BP =

B

SD-LCN	SD-SCN	SD	SB
1	0	0	0
0	1	0	0
1	0	0	0
0	1	0	0
2	1	1	0
4	2	2	0
2	1	1	0
6	3	3	1
4	2	2	1

P

Jan	Feb	Mar
100	125	140
150	180	200
0	0	0
0	0	0

=

L

Jan	Feb	Mar	
100	125	140	SD-LCN
150	180	200	SD-SCN
100	125	140	CN-LG
150	180	200	CN-SM
350	430	480	SD
700	860	960	SB
350	430	480	NUT
1050	1290	1440	SF
700	860	960	IM

Figure 1.5. Resource requirement for SD-LCN and SD-SCN in the production plan at the first quarter

$\underline{u}^t\mathbf{P}=$

.65	.33	.30	.12

100	125	140
150	180	200
0	0	0
0	0	0

=

Jan	Feb	Mar
65.0	81.3	91.0
49.5	180.0	200.0
0	0	0
0	0	0

Figure 1.6. Total ingredient cost of each product in each month

CHAPTER 2

APPLICATION OF THE PRODUCTION PLANNING FRAMEWORK TO A HYPOTHETICAL DAIRY PROCESSOR, CHEESE MANUFACTURER - PART I CHARACTERISTICS OF DAIRY PROCESSORS, AND CONFIGURATION AND ASSUMPTIONS OF HYPOTHETICAL CHEESE PLANTS

The dairy processing industry is the third largest sector of the food and beverage processing industries in terms of value of shipments. The 1987 Census of Manufactures indicated that dairy processors' value of shipments was $44.78 billion, the value added $11.89 billion, and the number of employees 141,200 (112). The Census statistics divided dairy products into five sectors: 1. creamery butter, 2. cheese (natural and processed), 3. dry, condensed, and evaporated dairy products, 4. ice cream and frozen desserts, and 5. fluid milk. The 1982 and 1987 Census statistics for the five sectors of the dairy products are summarized in Table 2.1. In terms of value of shipments, fluid milk is the largest sector followed by cheese, dried products, ice cream and frozen desserts, and creamery butter. The percentage changes in the value of shipment range from 27.1 percent gain for ice cream and frozen desserts to 15.8 percent drop for creamery butter. Consumer expenditures for the dairy products represented

12.5 percent of all consumer food expenditures (9). Consumption of dairy products is relatively stable because of slow change of consumer demographics, very long product life cycles, and few new product introductions (5, 52, 73).

Raw milk is the principal ingredient for dairy products and considerably influences the prices of the dairy products. Raw milk consists of two major solids components: fat and nonfat solids. While some dairy products are mainly made from a component like butter from the fat and NFDM from the nonfat solids, other products such as cheese and whole milk dairy products utilize both fat and nonfat solids components. Butter and NFDM are complementary products, and often made in the same plant or a multiplant company. The dairy products compete with each other for the overall milk supply (70). Milk is produced in every state, but half of the 1983 total was produced in five states: Wisconsin, Minnesota, New York, Pennsylvania, and California (35). Processed dairy products are manufactured from either surplus Grade A or manufacturing grade milk. There has been however a continuous decline of the manufacturing grade milk in the proportion of usage (68). Owing to the highly perishable nature, milk should be sold promptly in liquid form or processed into storable manufactured products. According to the perishable nature, the dairy products are divided into three types (9): 1) fluid milk (whole milk, low fat milk, skim milk, flavored milk drinks, and juices), 2) perishable manufactured products (cream, yogurt, ice cream, cottage cheese, and cultured products), 3) storable manufactured products (cheese, butter, dried dairy products, and canned dairy products).

Characteristics and Current Trends of Dairy Processors

The characteristics and current trends of dairy processors are analyzed to study their implications for production planning and control. The major characteristics and trends of the dairy processors are summarized below.

Seasonality of Production and Consumption

Production and consumption in the dairy industry are highly seasonal in nature. Milk production reaches a peak in the late spring and rapidly declines to a low in the fall. In contrast, the demand for fluid milk products is countercyclical to milk production, which is low in the spring, and high in the fall and winter. Accordingly, the milk price which is a primary factor in determining the manufacturing costs of dairy products fluctuates on a seasonal basis (67). Milk price is high from October to January and low from March to July. The highly seasonal milk production and price, and the demand for fluid milk products counterseasonal to milk production have led to the need for substantial amount of manufacturing capacity for storable products such as cheese, butter, and dried dairy products from the surplus milk in the spring. In contrast, dairy processors, particularly cheese and butter manufacturers, have difficulties in obtaining adequate supplies of milk in the fall. Also, the irregular fluctuation of milk production during any season causes regular difficulties for dairy processors. Thus, storable dairy product manufacturers face the problems of managing highly variable, uncertain volumes and prices of milk, variations in plant utilization, operating schedules and costs, and marketing and pricing. The competitive position of the dairy processor primarily depends on its ability to obtain milk supply enough to efficiently manage a material flow and fully utilize equipment and labor.

Declining Number of Dairy Processors but an Increase in Plant Size

Decline in the number of plants, and increase in plant size and productivity are major trends in the dairy industry, as shown in Table 2.1. Coupled with the market interventions by the government, increase in plant size and productivity over time has influenced on reducing processing costs and constrained dairy product price increases. While the number of U.S. dairy processors has been declining, the average annual throughput for the plants has increased greatly over time (113). Although large plants still represent a small proportion of total plant numbers, they account for a sizable proportion of volume processed (46). The decline in plant numbers does not result in a decline in competition, because the larger firms compete in much larger sales areas. It is expected that the number of dairy processors would continuously decrease with increasing scales of production coupled with the large investments in equipment and technologies in the foreseeable future.

Advancement in Mechanization and Technology

The reduction in the number of plants and the increase in the production scale have been accompanied by the mechanization of the operations, the rapid adoption of new technologies, and the implementation of cost-saving techniques during the last two decades (85, 96). In the past, cheese making has been an art rather than a science and very labor-intensive. Today, cheese processing is moving toward automation and more consistent quality. However, the advent of mechanization has not changed the basic principles of cheese manufacture. Mechanization of Cheddaring and hooping reduced labor requirements by 40 percent (50). With increased market size and higher levels of production capacity, considerable progress has been made in process automation as a means of reducing production costs. For plants processing more than 200,000 pounds of milk per day, unit processing costs may be reduced by automating some of the production processes (9). The mechanization of the operation would continuously contribute to improving manufacturing efficiency and unit cost through better product consistency, reduced energy and material losses, and reduced labor requirements. The disadvantages in economies of manufacturing scale and facilities may force old, small plants to close or improve manufacturing cost-efficiency by mechanizing the operation or switching the product mix fit for their scales and facilities.

Diversification of Dairy Processors

Diversification into other food business has been a major trend among the large processors which once were associated primarily with the dairy industry. The low profitability and generic nature of the business has led national dairy processors to gradually withdraw from the dairy business, especially fluid milk and bulk manufactured commodity operations such as cheese. National dairy processors have a strong presence in storable products, but their focus is on packaging and marketing finished products rather than on the production of bulk commodities such as cheese and butter (9). Today, most of national dairy processors produce many nondairy product lines.

Location of Dairy Processors

The location of processing facilities has a great influence on the growth and profitability of the dairy processor. Due to the perishability and bulkiness of raw milk and finished products, the location of milk production and processing plants are extremely important to the dairy processors. For example, greater raw milk availability makes dairy processors more profitable in the Upper Midwest. While fluid milk processing plants are generally located closed to consumption areas because it is cheaper to transport raw milk than finished fluid products, cheese plants are traditionally located near the source of milk supplies because it would have advantages in availability and reliability of milk, and permits considerable reduction in transportation and storage charges. Thus, milk used for the cheese manufacture generally moves a short distance to the plant, in contrast to milk used for fluid purposes which may go hundreds of miles. Other factors that affect the choice of location are proximity to markets, transportation costs, and dependable supplies of skilled labor and utilities. Changes in economics, institutions, technology, and transportation have resulted in larger, more widely spaced plants, however. For example, a shift in milk assembly from cans to bulk tank pickup has greatly expanded the distance over which milk can be moved from farms to plants (24). Many plants nowadays obtain milk supplies from producers and cooperatives, other plants, and receiving stations in broad areas.

Impact of Regulations and Policies on Dairy Processors

The dairy industry is affected by several regulations and public polices. The dairy price support program has the objectives to assure an adequate supply of milk, establish prices reflecting changes in production cost, and assure a level of farm income which will maintain needed production capacity. Additionally, milk marketing orders support the stabilization of prices and marketing. These dairy support programs reduce risks faced by farmers, and result in greater production at a given price level and less price differences among manufactured dairy products.

These government interventions are expected to be tempered or eliminated in order to avoid excessive government stocks and costs (48). If the government interventions are reduced, the higher price risks would work to the disadvantage of both dairy farmers and consumers. Processors would also face greater risks from greater fluctuations of raw milk supplies, and more volatile raw material costs and finished product prices. In addition, the reduction in the price support program would result in a drastic increase in competition among dairy processors for dairy product sales, and eventually more fluctuating and lower product prices (46).

Dairy processors need to be more conscious about changing costs of raw materials and products, and need to maintain profitability by flexibly changing among raw materials to sustain a desired level of profits. The dairy processors producing multiple products may more efficiently handle the fluctuating production and demand by producing the products with the competitive advantages. The dairy processors which efficiently manage the production plan and product pricing in response to the changes in market conditions will gain apparently greater production and marketing advantages as the market intervention by the government would reduce.

Flexible Product/Plant Portfolio

The product demands which are subject to uncertainty and fluctuation coupled with seasonal supplies of raw materials would lead to a growing demand for a plant or plant portfolio which can accommodate the manufacturing of different types and sizes of dairy products. Many fluid milk processing plants process cream, cottage cheese, ice cream, yogurt, and package fruit juices and flavored drinks. Cooperatives are heavily involved in processing and marketing of fluid milk and storable dairy products (100).

The burdens of volatile milk production and reduction in government intervention will fall mainly on processors of storable dairy products such as cheese and butter because the remainders of raw milk are used to make these products and capacity utilization become more volatile. It is costly to operate the plants processing storable products under conditions where output fluctuates greatly. In cheese manufacture, the need for a flexible plant portfolio producing multiple products is expected to increase. This will more efficiently handle fluctuating supplies and meet the increasing demand for more varieties and different sizes of cheeses. It will also improve plant utilization. Dairy processors need to be more conscious about changing costs of raw materials and products, and need to maintain profitability by flexibly changing among raw materials to sustain a desired level of profits. The marketing advantage of being able to switch product mix in response to price differences apparently has been more than offset by economies of production because price differences among dairy products have been restrained by government intervention through the price support program. The termination or reduction in market intervention by the government would provide greater incentives to the more flexible manufacturing plants as the cost efficiency of flexible product and price management would increasingly exceed the manufacturing inefficiency of maintaining multi-product production facilities. The dairy processors which efficiently manage the production plan and product pricing in response to the changes in market conditions will apparently gain greater manufacturing and marketing advantages as the market intervention by the government would reduce.

Configuration and Assumptions of Cheese Plants

It is obvious that an organized production planning framework provides management with greater control over its business to improve profitability and productivity. Craig et al. (27) reported an economic advantage of producing natural cheese and process cheese food in one plant by applying a linear programming model. Even though most of process cheese plants do not manufacture the natural cheese in the same plant, the integration advantage of natural cheese and process cheese production systems can be extended to separate cheese plants under a company, or other food plant portfolio such as a fluid milk processor manufacturing frozen products or a food processor manufacturing multiple products in the same plant.

Mathematical optimization and matrix theory are suggested in chapter one as useful bases for a production planning framework which addresses the food processor's typical characteristics and problems. To illustrate how the production planning framework is built to provide management with decision support information and to validate the potential use of the framework, an example of a hypothetical dairy processor's Cheddar and process cheese plants is used. Although the example is hypothetical, it contains many of the elements of a real manufacturing environment. This example is used in later chapters to illustrate the application of mathematical optimization and matrix theory in the food industry settings. Considering the seasonality of milk production and dairy product consumption and changes in input resource costs, organized production planning can help the dairy processor enhance the profitability by allocating the milk supply to the products based on the comparative cost or profitability of the products, by balancing the supply and demand of the products through the optimization of formulation and product mix, and by controlling material flows. We presented configurations and consumptions of the cheese plants to help readers understand the background situation of the production planning framework application example.

It is assumed that the Cheddar cheese plant manufactures two types of Cheddar cheeses, 40-pound block and 500-pound barrel, and by-products: cream, whey cream and condensed whey. The process cheese plant manufactures the process cheese food and five different flavored process cheese spreads by utilizing barrel Cheddar and whey products of the Cheddar cheese plant. The production planning framework is built based on the following configuration and assumptions of the cheese plants. Table 2.2 shows the brief configuration of Cheddar and process cheese plants.

Plant Operation

Like other food processors, dairy processors rely on high volume capacity because of a low profit margin for maximizing the long-run profitability. The factors affecting the profitability of the cheese manufacture include milk supply, input resource costs, cheese sales and stocks, plant scale, capacity utilization, and prices for other dairy products in competing for milk. When the market prices of other dairy products are strong, for instance, it is not easy to get milk away for cheese manufacture from manufacturing the other dairy products.

Dairy processors' operating standards range from continuous operations (24 hours per day, 7 days per week) often found in large plants to a normal workweek (8 hours per day, 5 days per week) in some smaller plants. The Cheddar cheese and process cheese plants are assumed to operate with a schedule of 7 days per week and 24 hours per day. The Cheddar plant operates with a total processing time of 20 hours (including milk filling time of 18 hours) and a cleanup time of 3 hours per day, while the process cheese plant with a total processing time of 20 hours and a cleanup time of 4 hours per day. The equipment is intensively utilized and raw milk storage costs are kept to a minimum by working seven days a week. The operating schedule of the plants may vary in order to adjust to the changes of the demand, milk supply, market conditions or other factors affecting the operation. In the periods of excessive milk supply or demand, the plant may top normal manufacturing practices for a short time by running equipment beyond its capacity, slightly shortening the cheese making time, or running more hours at the expense of cleaning time. The change in schedules will lead to the changes in the manufacturing costs per pound of cheese due to the variation in average labor and utility costs. In general, the average manufacturing costs decrease with an increase in the plant capacity utilization, while the costs increase as the plant operates beyond full capacity. The framework is built based on a full capacity utilization. Production planning is presumed to be carried quarterly and minor planning monthly or weekly to adjust to variations or discrepancies between actual and planned production. The material flow charts of the cheeses are shown in Figures 2.1 and 2.2.

Cheddar Cheese Plant

Cheddar cheese is the most important single variety of cheese in the world. While Italian-style cheese sales are growing fast, Cheddar cheese remains the most popular cheese in the U.S. (3). Cheddar varieties and substitutes made up 46 percent of the value of cheese sold in food stores in 1987. Cheddar cheese is defined as containing not more than 39.0 % moisture, and not be less than 50 % of the fat as dry matter (FDB) under Federal Standards of Identity. The yield of Cheddar cheese on the average is 9.05 to 10.27 pounds per 100 pounds, but different types of Cheddar have different yields (60, 76). Wilster (119) enumerated the criteria for desirable quality attributes of Cheddar cheese in terms of texture, flavor, and slicing property.

Cheddar cheese manufacture can be classified into two broad categories: block Cheddar cheese and barrel Cheddar cheese. These two groups are slightly different in the manufacturing process and production economies. However, it is difficult to make a clear statistical differentiation between two groups because most statistics report Cheddar cheese information only as one group (69). While block Cheddar is a more consumer-oriented product, barrel Cheddar is mainly used as a raw material in other processes such as process cheese varieties. In the manufacturing process, barrel Cheddar cheese is not pressed after salting, but ripened as curds in large polyethylene-lined drums or barrels. Block Cheddar cheese is made for market and sold at higher price than barrel cheese, while the cost of barrel cheese is usually lower because of labor savings. The weights of block Cheddar are 20, 40, 60, and 640 pounds, but 40 and 640 pounds are the most common. These blocks are usually cured and then cut into retail packages. Barrel Cheddar is a 500-pound barrel made of either fiber or steel. Even though barrel Cheddar is often used for further processing into processed cheese products, it may be carved and cut into retail-size packages (8).

Processing capacity

The Cheddar cheese plant is assumed to have a capability of manufacturing 40-pound block and 500-pound barrel Cheddar cheeses. The plant supplies the barrel cheese to the process cheese plant and the block cheese to the natural cheese market. Barrel Cheddar is used for manufacturing the process cheese varieties in the process cheese plant because flavor and body develop faster, and the deletion of pressing stage and shorter cooking and cheddaring reduce the production costs. The high moisture content (38%) of the barrel Cheddar produces high cheese yield. The production capacity of Cheddar cheese is 720,000 pounds of milk per day. When an average yield of Cheddar cheese is assumed 10.0 pounds per 100 pounds milk, such amounts are comparable to 72,000 pounds of natural cheese per day and 2,160,000 pounds of Cheddar cheese per month. The milk supply is assumed to have an average milkfat content of 3.7 percent.

Milk holding capacity

Raw milk is supposed to be delivered to the plant every day. The daily holding capacity of milk is assumed as 900,000 pounds, which is greater than the daily processing capacity of the plant. The holding capacity as percentage of milk processed per day is 125%. This holding capacity provides the plant management with flexibility enough to efficiently schedule the operation in relation to the changes of milk supply and market conditions. Pasteurizers, vats, and milk silos are the places where manufacturing bottlenecks are most likely to happen.

Capacity of the pasteurizer

A pasteurizer is one of the major measures for the cheese plant capacity because all of the milk must pass through it (68). The pasteurizer is arranged for continuous milk flow with the vats. It determines how fast the vats can be filled since the vats are filled in succession. The capacity of the pasteurizer is measured in pounds of milk processed per hour. The pasteurizer capacity of the plant is assumed 40,000 pounds of milk per hour, which leads to 720,000 pounds of milk per day as a maximum total daily milk volume, based on the 18 hour milk filling time. The pasteurizer is cleaned every 12 hours for maximum efficiency by C.I.P. system, with cleaning time of about 40 minutes in the plant.

Capacity of the cheese (cooking) vat

The plant has 6 cheese vats with 45-minute filling time per vat. It therefore takes 4.5 hours to fill all the vats one time. Each vat has a capacity of processing 30,000 pounds of (standardized) milk with a turnover rate of 4 times per day. The plant operates smoothly and continuously so that when one cheese vat is full, another becomes ready for filling until the last vat is made.

Whey processing

Whey contains most of lactose, salts and water-soluble proteins of the milk after casein and fat are separated as curds in the cheesemaking processing. Cheese manufacturers, however, have traditionally considered whey as a waste product for many years, because it had little economic value. Whey was usually either dumped in a sewer or stream, or used as pig feed or fertilizer. Enforcement of pollution standards, and awareness of the intrinsic nutritional and economic values of whey have made it less practical to treat whey as a waste product. Consequently, whey is now regarded as a valuable by-product (1).

Whey processing is an important additional operation in the plant. Unseparated whey is temporarily held in a whey silo, and then passes through a fines saver, a pasteurizer, and a separator. The separator can remove 45% fat cream for both sweet cream and whey cream. When whey cream is separated, it is assumed that whey fat is fully recovered. Then, the plant concentrates the separated whey (6.5% TS) to condensed whey (60% TS). Some large cheese plants sell whey in dry form. Dry whey is a powder consisting mostly of protein and milk sugar. Although it is primarily used in animal feeds, it is also used for various food products, hot chocolate and infant formula. When it is assumed that the plant daily produces 72,000 pounds of Cheddar cheese and generates 8.5 pounds of separated whey per pound of Cheddar cheese, the daily production of the condensed whey is about 66,298 pounds. The detail of computing the amount of condensed whey production and the production cost is given in Exhibit 3.2. Whey cream and condensed whey are assumed to be used in the processing cheese plant or sold in bulk and moved out of the plant regularly.

Storage capacity of Cheddar cheeses

Product storage capacity is regarded as a capacity constraint for production. The plant is assumed to have the storage capacity equivalent to 30-day production. Particularly, the barrel Cheddar cheese will be stored for average 30 days before being moved to the process cheese plant. Storage cost is assumed $0.2 per 100 pounds of cheese for 30 days. Storage capacity for aging the cheese is not considered in the plant. The final aging, grading and washing operations are performed at distribution centers operated by cheese marketing organizations.

Process Cheese Plant

Process cheese is made by blending and heating of several lots of natural cheese with suitable emulsifiers into a homogeneous plastic mass (77). Process cheese can be made from most varieties of cheese, but Cheddar cheese is most commonly used (115, 270). A blend of different lots of natural cheese of various ages, physical properties and compositions is selected to obtain desired composition and physical and flavor characteristics of the process cheese. The advantages of processing are convenience, uniformity, longer keeping quality (flavor and body), melting quality, and various flavor and packaging options (77, 89, 107).

Process cheese may contain fruits, vegetables, meats and spices. The natural cheese is not only the most important but also the most expensive ingredient in the process cheese manufacture. Natural cheese usually accounts for 60 to 75 percent of the weight of the process cheese. The age, composition, acidity and flavor of natural cheese directly influence those of the process cheese (25, 60, 83, 106). Thus, process cheese manufacturers must be familiar with the quality, characteristics, economic consideration of the natural cheese used for processing to maintain consistent product quality.

Some plants use inferior young cheese with broken or damaged rinds or putrid cheese before flavor defects are developed (56). But the natural cheese from which the process cheese originates is not generally of undergrade quality, or culls as is often erroneously assumed (60). Quality process cheese can not be produced without quality natural cheese. In reality, natural cheese with inferior quality comprises a very minor part of the natural cheese source. About 53% of domestic natural ripened cheese is made into process cheese products. Some process cheese manufacturers expand by increasing production, adding process cheese product lines, or manufacturing natural cheese (90). It would be worthwhile to build factories wholly devoted to the production of good quality natural cheese destined for the pasteurized process product (60).

There are three general types of processed cheese defined under the Food, Drug and Cosmetic Act: pasteurized process cheese, pasteurized process cheese food, and pasteurized process cheese spread (114). These differ in moisture and fat contents, and in number and kinds of food ingredients allowed. Federal Standards of Identity state that the maximum legal moisture of process cheese is 40 percent and the minimum fat in the dry matter is 50 percent. Process cheese food must contain not more than 44 percent moisture and not less than 23 percent milk fat, whereas process cheese spreads must contain not less than 44 percent and not more than 60 percent moisture, and not less than 20 percent milk fat.

Process cheese food requires the same selection, trimming, grinding, heating, and emulsification principles used for pasteurized process cheese. The process cheese food has a softer body and milder flavor than the process cheese, however. The process cheese food contains higher moisture and less fat, and is made by higher cooking temperatures, and lower pH. Higher heat and lower pH provide greater protection against most bacterial spoilage. The pH range of most process cheese foods is usually 5.6 to

5.4, but may show a lower limit of 5.2. Also, optional ingredients which are not permitted for process cheese are used.

Process cheese spread is made by manufacturing principles conforming closely to those of process cheese food, except more moisture and lower fat. Process spread is allowed the same optional milk ingredients as process cheese food, but in addition, carbohydrates such as corn syrup solids, starches, sugars, and gums like carob bean, gelatin and algin, not to exceed 0.8% by weight, may be used (60). Higher moisture gives a spreading quality to the product, but results in greater bacterial activity (84). Therefore, the cooking temperatures are very high and the pH low.

Processing capacity

The plant is assumed to manufacture process cheese food, and the following five process cheese spreads: cheddar, chives and onion, nacho and red pepper, bacon and hickory smoke, and salami and hickory smoke. One month old young barrel Cheddar cheese from the Cheddar cheese plant is used to manufacture the process cheese varieties in the process cheese plant. The use of barrel Cheddar is advantageous to the process cheese plant due to fast development of flavor and body, and its high cheese yield. The production capacity of process cheese products is assumed 50,000 pounds per day. The production of each product varies with the demand for the product. When an average use of Cheddar cheese for the process cheese products is 50.0 pounds per 100 pounds products, and 70 percent of Cheddar is young aged, 25,000 pounds of Cheddar cheese are needed for the process cheese manufacture and 17,500 pounds of young barrel Cheddar are needed from the Cheddar cheese plant a day.

Cheese selection and blending

The selection of natural cheeses for blending is the most important step in processing cheese. Cheddar cheeses are selected according to source, flavor, acidity, age, body and texture. It is difficult to obtain all the desired qualities in one cheese. Thus, a blend of different lots of cheese is selected to obtain desired composition and physical and flavor characteristics. When cheese is received, condition, age, source, taste, flavor, and physical properties are checked with the arrival date for blending. The cheeses received are placed in cold storage at a temperature of 4 to 5 °C to minimize further maturation unless they are used immediately.

The batch size for a blend is assumed as 10,000 pounds. The cheese blend batch for process cheese food accounts for

15 percent young Cheddar, 25 percent medium-aged Cheddar, and 60 percent old-aged Cheddar, while the cheese blend batch for process cheese spread 15 percent young Cheddar, 15 percent medium-aged Cheddar, and 70 percent old-aged Cheddar. Young Cheddar cheeses are received from the Cheddar cheese plant and average 1 month old. Medium-aged and Old-aged Cheddar cheeses are purchased from the market. Medium-aged Cheddar cheeses are 3 to 5 month old and old-aged Cheddar cheeses 6 to 9 month old.

Capacity of the process cheese cooker

The plant has 5 cookers with a turnover rate of 5 times per day. Each cooker processes 2,000 pounds of process cheese ingredients. The plant operates smoothly and continuously so that when one cheese cooker is full, another becomes ready for filling until the last cooker is made.

Packaging of process cheese products

Packaging is a part of value-added manufacturing. Each product in the process cheese plant has two types of packaging units: a case of 50, 8 ounce cups and a case of 25, 16 ounce cups, which lead to 12 different packaged process cheese products. Cardboard boxes lined with a suitable wrapper are used for all sizes.

Definition and Components of Production Costs

The total product costs include manufacturing costs and general expenses. The optimization of cheese manufacture will consider only direct production costs. The direct production costs include costs of raw materials, utilities, labor and production supplies as a part of the manufacturing costs. The objective function will be defined as the maximization of total profit contributions of manufacturing the products or the minimization of total direct production costs. In the optimization, a profit contribution per unit of a product is determined by selling price minus direct production cost per unit of the product. To find a satisfactory product mix is one of the objectives in the production planning framework. However, it is true that the optimized product mix will not provide the real net return the firm can earn since it does not take fixed costs into account. It is not practical to allocate the fixed costs to each product before determining the product mix. It would not be unreasonable to exclude the fixed costs because the fixed costs can be added and then the real net return can be determined after the solution is attained.

The components of the direct production costs for Cheddar cheese and process cheese products are described as follows:

Raw Materials

Costs of raw materials such as production ingredients and packaging materials are the major costs in the dairy industry. The ratio of the cost of raw materials to total product cost apparently varies for different types of products or sizes of the plants. It is assumed that the use of raw materials changes proportionally with the production of cheese.

Labor

Labor is one of the most important components of the production costs in the cheese plants. Labor costs vary broadly, depending on different production labor requirements or productivities caused by various plant sizes, cheesemaking technologies, plant designs, labor polices, and so on. In determining the labor costs consideration must be taken for the type of labor, prevailing wage rate, and labor productivity.

The production labor includes people involved in the receiving, cheesemaking, storage, whey processing, cleaning, laboratory testing, and maintenance. The production labor costs are divided into supervisory salaries, indirect labor costs, and direct labor costs. Supervisory and indirect labor include plant manager and other supervisory personnel, plant guards, truck operators, and laboratory technicians. Direct labor is all labor that is obviously related to the production. The direct labor costs are supposed to proportionally vary with the production of cheese. Since the framework considers solely direct production costs, the indirect labor cost is excluded in the framework.

A wage rate of $10.00 per hour is used for all direct labors. The wage rate is an average for various labor types, overtime charges, night and holiday payments. Production labor productivities (pounds of cheese per hour of direct variable labor) for the Cheddar plant and process cheese plant are supposed 169.4 pounds and 250 pounds, respectively. Daily labor requirements for the cheese and whey processing of the plant are assumed 425 hours with a 24-hour operating schedule, while those for the process cheese plant are assumed 200 hours. Therefore, an average labor cost per 100 pounds of Cheddar cheese is $5.69, whereas an average labor cost per 100 pounds of process cheese products $4.00.

Utilities

Dairy processors generally use large amounts of water for washing, cooling and steam generation, and manufacturing as a raw material (76). The power and steam requirements are also high in the dairy industry, and electricity and fuel are ordinarily required to supply these utilities. Energy consumption is a growing item in overhead expenses in cheesemaking. Recovery of heat used during processing is crucial to the utility cost-saving. Utility costs vary broadly depending on the amount of consumption, plant location, and source of utilities. The utilities are usually used for the production of several different products; thus, the utility costs are apportioned among the different products based on the amount of individual consumption.

The utilities used in the plants are assumed electricity, natural gas, water and sewage. Electricity is charged at a flat rate of 6.5 cents per kilowatt hour (KWH). Electricity requirements for the Cheddar cheese plant with a 24-hour operating schedule is assumed as 6,800 KWHs. Thus, an average electricity cost per 100 pounds of Cheddar cheese is $0.58, while an average electricity cost per 100 pounds of process cheese is $0.43. Natural gas rate of 45 cents per therm is used for steam generation. Natural gas requirements per day for the Cheddar cheese plant and the process cheese plant with a 24-hour operating schedule are assumed 1,020 therms and 600 therms, respectively. Thus, an average natural gas cost is $0.61 per 100 pounds of Cheddar cheese, and $.54 per 100 pounds of process cheese. Water consumption per 100 pounds of Cheddar cheese and process cheese is assumed 12 gallons and 8 gallons, respectively. No direct charge is made for water since the plant is assumed to have its own water well with an unlimited water supply.

Many legal restrictions are placed on the disposal methods for waste materials from the process industries (76). The plant has adequate capacity and facilities for correct waste disposal. Sewage cost of $1.20 per 1,000 gallons of fluid disposal in the sewage system is used. The amount of fluid disposal in the plants is assumed 60,000 gallons of the Cheddar cheese plant and 33,000 gallons of the process cheese plant. Therefore, the water and sewage cost per 100 pounds of Cheddar cheese is $0.09, while that per 100 pounds of process cheese is $.08. Accordingly, total average utility costs are $1.26 per 100 pounds of Cheddar cheese and $0.978 per 100 pounds of process cheese products.

The utility costs of the Cheddar cheese plant do not include the utility costs of whey processing. Whey cream and condensed whey are important as not only process cheese ingredients, but also additional revenue sources to the plant. Revenues and costs of whey processing are therefore ascribed to the cheese manufacturing process. Whey processing for whey cream and condensed whey adds $0.01 and $1.64 per 100 pounds of Cheddar cheese to the cheese production cost, respectively.

Maintenance

A considerable amount of expenses is needed for maintenance if a plant is to be kept in efficient operating condition. The maintenance costs for the equipment are considered variable with the volume of milk processed at the plant. Maintenance cost is charged at a rate of 10 cents per 1000 pounds of milk and 100 pounds of process cheese products.

Production supplies

Many miscellaneous supplies are needed to keep the process practices efficiently. Items such as test chemicals, cleaning supplies, and custodial supplies can not be considered as raw materials or maintenance materials. The cost for production supplies is charged at a rate of 45 cents per 100 pounds of Cheddar cheese and process cheese.

Table 2.1. Census statistics for the dairy products[a]

Dairy products industry	SIC code	Value of shipments ($000,000)	Value added ($000,000)	Number of employee (000)	Number of firms
Creamery butter	2021	1420.2 (1686.8)	155.5 (135.6)	1.7 (2.2)	42 (61)
Cheese	2022	12926.1 (10762.8)	2612.1 (1777.3)	32.9 (29.6)	476 (575)
Condensed/ evap. milk	2023	5832.0 (4730.7)	2382.2 (1447.6)	14.0 (12.2)	125 (132)
Ice cream/ frozen des.	2024	3914.6 (2855.1)	1262.3 (910.4)	20.3 (17.8)	461 (482)
Fluid milk	2026	20690.4 (18736.0)	5478.6 (4088.9)	72.3 (78.2)	641 (853)
Total		**44783.3 (38771.4)**	**11890.7 (8359.8)**	**141.2 (140.0)**	**1745 (2103)**

( ): statistics of 1982 Census of Manufactures

[a] SIC: Standard Industrial Classification

Value of Shipments: received or receivable net selling values, f.o.b. plant of all products shipped as well as all miscellaneous receipts.

Value added by manufacture: the value of shipments minus cost of materials, supplies, utilities, and contract work.

Employees: all full-time and part-time employees on the payrolls at any time during the year.

Source: 1987 Census of Manufactures, U.S. Department of Commerce Bureau of the Census, Washington, DC 20233.

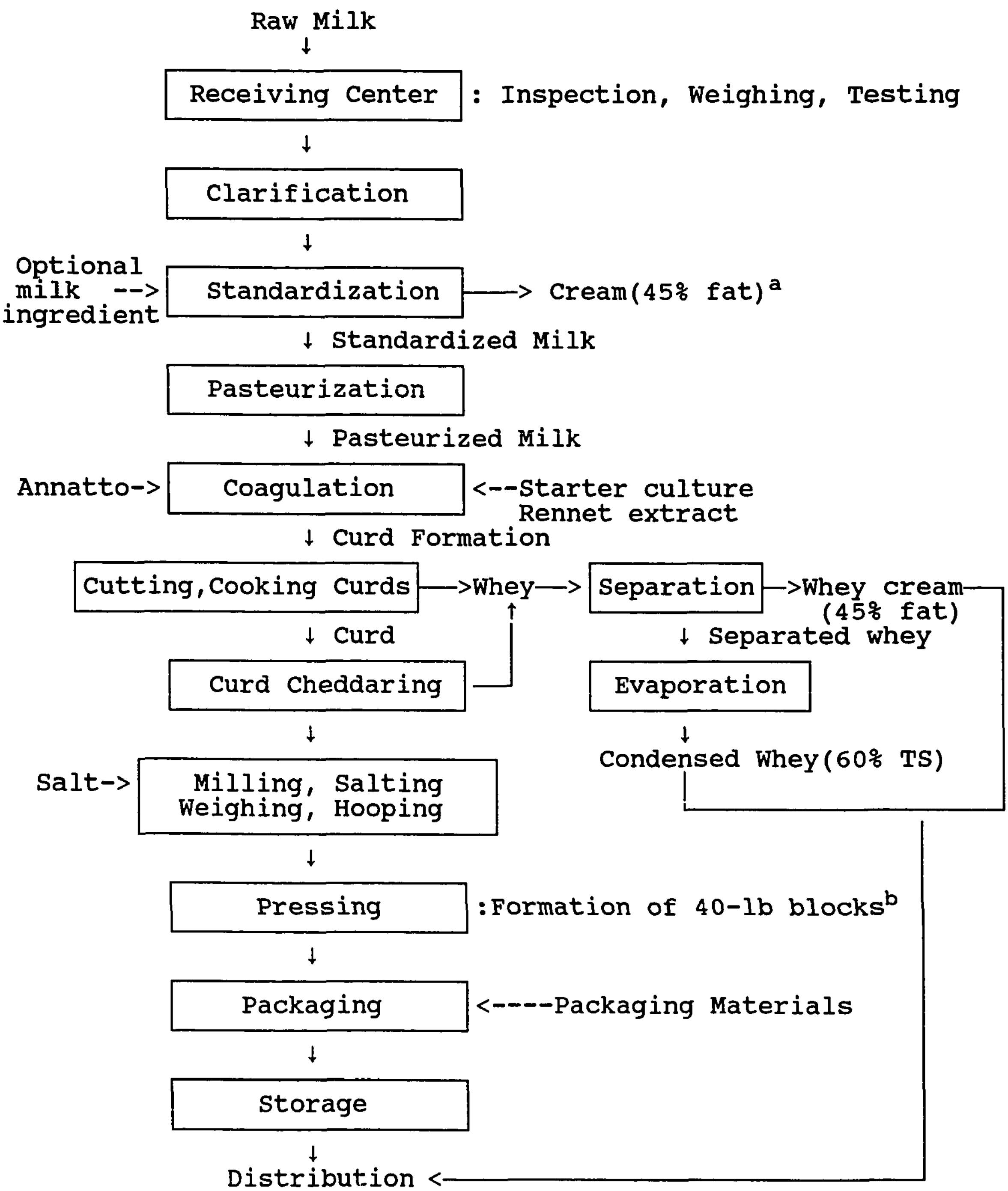
Table 2.2. Configuration of Cheddar and process cheese plants for optimizing production planning models

Item\Plant	Cheddar cheese	Process cheese
Plant operation	7 days/week 24 hours/day	7 days/week 24 hours/day
Daily processing capacity	720,000 lbs milk	50,000 lbs products
Products	40 lb block 500 lb barrel cream[c] whey cream condensed whey	process cheese food[a] process cheese spread[b]
Storage capacity	30-day production	50-day production
Batch process and capacity	6 cooking vats (30,000 lbs milk capacity)	cheese blend (10,000 lbs) 5 cheese cookers (2,000 lbs capacity)

[a] Process cheese food includes 2 packaging lines: a case of 50, 8 oz. cups, and a case of 25, 16 oz. cups.

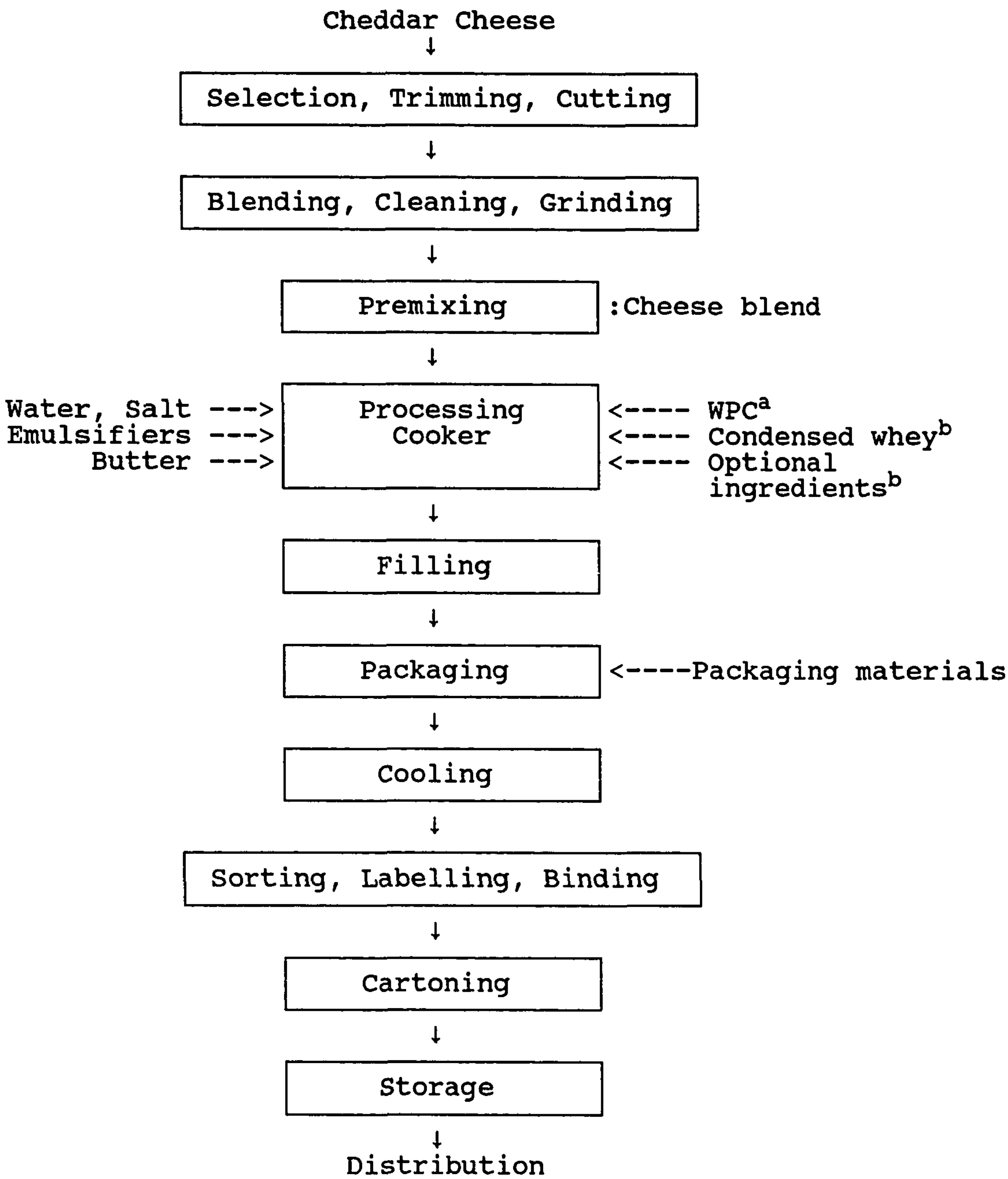
[b] Process cheese spread includes 5 different flavor types of spreads with 2 packaging lines.

[c] Production of cream depends on the standardization of milk.



[a] Production of cream depends on the standardization.
[b] Barrel Cheddar does not include the pressing stage.

Figure 2.1. Production Flow Chart of Cheddar Cheese



[a] Whey protein concentrate (WPC) is an ingredient of process cheese food
[b] ingredients of process cheese spreads

Figure 2.2. Production Flow Chart of Process Cheese Products

CHAPTER 3

APPLICATION OF THE PRODUCTION PLANNING FRAMEWORK TO A HYPOTHETICAL DAIRY PROCESSOR, CHEESE MANUFACTURER - PART II MATHEMATICAL OPTIMIZATION AND MATRIX THEORY APPLICATION

A production planning framework is developed for a Cheddar cheese plant and a process cheese plant of a hypothetical dairy processor. Optimization of mathematical models and matrix theory approach are suggested as foundations of the production planning framework. Figure 3.1 describes a scheme of generating production information in the production planning framework. This chapter discusses the mathematical optimization application and the matrix theory application to production planning. While mathematical optimization is used to optimize Cheddar cheese formulation and product/batch mix, matrix theory is used for building a bill of materials (BOM), and organize and manage logistics information through Matrix data structures (MDS).

Optimization of Cheddar Cheese Formulation

Optimization models are constructed to find block and barrel Cheddar cheese formulations or recipes. Assumptions, decision variables, objective functions and constraints for the models are explained as follows:

Assumptions

Cheddar cheeses manufactured in the plant are block and barrel cheeses. While block Cheddar cheese is made for market, barrel Cheddar cheese for further processing into processed cheese products at the process cheese plant of the dairy processor. Supplies of input resources are assumed limitless except raw milk. Composition, unit of measure and unit cost of input resources available for use are presented in Table 3.1. Weights of block and barrel Cheddar are 40 and 500 pounds, respectively. Whey cream and condensed whey are manufactured as by-products from whey at the Cheddar cheese plant. Cream may be removed to be sold as a financial source or added as an ingredient depending on the optimization of raw milk standardization. Standardization of milk by adding solids-not-fat (SNF) or removing milk fat is crucial to meet manufacturing standards, maintain uniformity of cheese quality, and obtain a maximum efficiency in the use of incoming materials. It is assumed that fat contents of cream and whey cream removed are 45 percent. Milk fat and whey fat are supposed to be recovered with a yield of 100 percent.

When cream is removed from raw milk, nonfat substances of the milk will be also removed. Among the nonfat substances, casein is the most important milk solid for cheese yield. The amount of casein removed must be therefore taken into account to correctly measure the yield of cheese from standardized milk. When the fat and casein contents of the raw milk are assumed 3.7 percent and 2.58 percent, the casein content of the removed cream is 2.68 percent as follows:

$$\frac{C}{100 - F} = \frac{2.58}{100-3.70}$$

$$= .0268 \text{ lb casein/lb nonfat substances of raw milk}$$

where

F = fat percentage of raw milk;

C = casein percentage of raw milk.

Since the removed cream has 45 percent fat and 55 percent of the cream is a nonfat proportion of the milk, the casein content of the cream removed is 1.47 percent: .55(.0268) = .0147 lb casein/lb cream removed. Thus, 100 pounds of the cream removed contain 45 pounds of fat and 1.47 pounds of casein, while 100 pounds of cream purchased contain 45 pounds of fat and 1.39 pounds of casein. Despite different casein contents, the prices of the cream removed and the cream purchased are the same because the price is determined based on the fat content of the cream.

Table 3.2 lists prices, and fat and moisture contents of finished products and by-products available at the Cheddar cheese plant. For keeping the desired quality attributes of cheeses, moisture contents of block Cheddar and barrel Cheddar are assumed to remain at 37 percent and 38 percent, respectively. The aged Cheddar cheese with the highest quality was made at a moisture content up to 37 percent and MNFS up to 62 percent (63). Higher moisture content of barrel Cheddar leads to higher cheese yield at a lower cost. Moisture control of Cheddar cheese depends on the conditions at all stages of manufacturing: amount of starter and coagulant, cutting, heating, stirring, piling, washing, pressing, salting and curing. The 38 percent moisture content of young barrel Cheddar would not affect the quality of process cheese products since the barrel Cheddar cheese is used after one month storage, and mixed with aged natural cheese with desired quality attributes at the process cheese plant. The values used in the cheese yield formula are 1.09 salt solids retention factor, 93 percent fat retention and 96 percent casein retention (27, 59). Major constraints for the optimization model of cheese manufacture are given in Table 3.3. Exhibit 3.1 describes the computation of cheese, whey cream and separated whey yields associated with the cheese manufacture, and Exhibit

3.2 shows how the yield of condensed whey and the cost of condensing whey are derived.

Decision variables

Decision variables for the Cheddar cheese formulation optimization are identified with the amounts of the resources which may be used and the amounts of output products including the cream which can be removed during the standardization of milk. The decision variables are listed in Table 3.4. Formulations of block and barrel Cheddar cheeses were determined based on a unit of cheese vat because the ingredient proportion in an optimum formulation of a vat is the same as that of a multiple number of vats. The resources that are not used for the standardization have little influence on the cheese yield of a vat when the levels of their usages are controlled in acceptable ranges. For example, the effect of pH on Cheddar cheese quality is relatively small when the range of pH is between 4.9 and 5.4 (63). Amounts and costs of these input resources per cheese vat are fixed in the model, regardless of the cheese yield from a vat.

Objective functions

Maximization of the unit profit contribution margin of a product is chosen as an objective function for optimizing the formula (recipe) of block Cheddar cheese, while minimization of the direct production costs of a unit product is for barrel Cheddar because barrel Cheddar is internally used to manufacture process cheese products as an intermediate product. The formulations are chosen based on comparisons among optimal solutions of different objective functions described in Tables 3.5 and 3.6. Table 3.5 shows that maximizing profit contributions or profit margin produced the same solution as minimizing the production costs per pound cheese, but produced better solution than minimizing the production costs. Table 3.6 shows that minimizing the production costs per pound cheese produced better solution than minimizing the production costs.

A model maximizing total profit contributions from a block Cheddar cheese vat

Objective function:

Total profit contributions from block cheese vat outputs are computed by subtracting direct production costs from total projected revenues of a block cheese batch output. Unit cost of input resources and unit price of finished products and by-products are shown in Tables 3.1 and 3.2, respectively.

Total profit contributions from a block cheese vat output

= total revenues from a block cheese vat output - direct production cost of making a block cheese vat output

Total revenues from a block cheese vat output

= 1.3075*F(block) + .8235*F(crm-rem) + .7875*F(wheycrm) + .078*F(condwhey)

Direct production cost of making a block cheese vat output

= .015*F(block) + .0016*F(crm-rem) + .0016*F(wheycrm) + .0178*F(condwhey) + .1197*X(milk) + .8235*X(crm-add) + .81*X(nfdm) + .239*X(condskim) + 378.3

Thus, the objective function is:

Maximize Z (= Total profit contributions from a block cheese vat output)

Z = 1.3060*F(block) + .8219*F(crm-rem) + .7859*F(wheycrm) + .0602*F(condwhey) - .1197*X(milk) - .8235*X(crm-add) - .81*X(nfdm) - .239*X(condskim) - 378.3

where 378.30 = other direct production cost of processing 30000 pounds standardized milk.

<u>Constraints</u>:

1. Capacity of a cheese vat (batch size)

   X(milk) + X(crm-add) + X(nfdm) + X(condskim) - F(crm-rem) $\leq$ 30000

2. Acceptable range of fat in the dry matter (FDM), moisture in the non-fat substance (MNFS), and casein to fat (C/F) ratios of cheese milk

Fat in the dry matter (FDM) and moisture in the non-fat substance (MNFS) are more important than the absolute values of fat and moisture in determining the quality of Cheddar cheese. While FDM could be controlled by altering the casein to fat (C/F) ratio through standardization, fat content could not be controlled (63). Moisture content in cheese can be also controlled by changing the FDM level because FDM is a function of fat and moisture contents in the cheese as seen in the following formula:

$$FDM = \frac{F}{100 - M} .$$

Moisture in non-fat substance (MNFS) is also a function of fat and moisture contents in the cheese:

$$MNFS = \frac{M}{100 - F}$$

where

F = fat percentage of cheese

M = moisture percentage of cheese.

MNFS influences the cheese yield (66), and is used as an indicator for the relative amounts of moisture and casein in the cheese because non-fat substances in cheese are mainly moisture and casein (63, 64, 65). In the mechanized cheese plant, FDM is related to MNFS in a cheese, probably as a result of the relative inflexibility of procedures for moisture control (64). Thus, modifying FDM is an effective way of controlling MNFS in cheese as milk composition changes seasonally. The amount of moisture per unit of casein affects the cheese flavor by affecting the activity of microorganisms and enzymes responsible for ripening (63). MNFS was the most important parameter affecting the grade score of the cheese (87).

An acceptable range of casein to fat (C/F) ratios of cheese milk is set between 0.68 and 0.70. The FDM level of Cheddar cheese can be determined based on the FDM formula described below:

$$FDM = \frac{FR(F)}{[FR(F) + CR(C)]\ SR} = \frac{FR}{[FR + CR(C/F)]\ SR}$$

where

FR = fat retention percentage divided by 100

F = fat percentage of standardized milk

CR = casein retention percentage divided by 100

C = casein percentage of standardized milk

SR = salt solids retention factor.

In the formula the following variables are fixed at constant values: FR = .93 , CR = .96, and SR = 1.09.

When using the formulas of FDM and MNFS, MNFS can be expressed as a function of FDM and M:

$$MNFS = \frac{M}{100 - FDM(100 - M)} \quad .$$

Thus, the levels of FDM and MNFS can be determined from the C/F ratio and the moisture content of cheese. The C/F ratio range ensures the FDM level of Cheddar cheese between .533 and .539, and the MNFS level between .557 and .560 as follows:

$$.68 \le C/F \le .70$$

$$\Rightarrow \quad FDM = .539 \quad MNFS = .560 \text{ when } C/F = .68$$

$$FDM = .533 \quad MNFS = .557 \text{ when } C/F = .70$$

$$\Rightarrow .533 \le FDM \le .539, \quad .557 \le MNFS \le .560.$$

Casein and fat percentages of standardized milk are calculated based on the casein and fat contents of potential standardization resources described in Table 3.1.

C = - 1.47*F(crm-rem) + 2.58*X(milk) + 1.39*X(crm-add) +
28.00*X(nfdm) + 9.20*X(condskim)

$F = -45*F(crm\text{-}rem) + 3.70*X(milk) + 45*X(crm\text{-}add) + .00*X(nfdm) + .37*X(condskim)$

From casein and fat percentages of the standardized milk, lower and upper levels of C/F ratios are obtained as follows:

Lower level of casein to fat ratio ($C/F \geq .68$):

$$29.13*F(crm\text{-}rem) + .064*X(milk) - 29.21*X(crm\text{-}add) + 27.32*X(nfdm) + 8.9484*X(condskim) \geq 0$$

Upper level of casein to fat ratio ($C/F \leq .70$):

$$30.03*F(crm\text{-}rem) - .01*X(milk) - 30.11*X(crm\text{-}add) + 27.3*X(nfdm) + 8.941*X(condskim) \leq 0$$

3. Maximum amount of cream that can be removed from raw milk

Forty five percent fat cream is assumed to be removed from 3.7 percent fat raw milk. Thus, .0822 pound of 45 percent fat cream can be available from each pound of 3.7 percent milk: 3.7/45 = .0822. The amount of cream which can be removed from the raw milk is described as follows:

$$F(crm\text{-}rem) \leq .0822*X(milk).$$

4. Cheese yield per batch

Cheese yield per 100 pounds of input resources is computed based on 37 percent of cheese moisture content and a cheese yield formula (27, 59) described in Exhibit 3.1, and incorporated into the following equation computing the total amount of cheese from a batch:

10.2387*X(milk) + 74.7159*X(crm-add) + 48.1157*X(nfdm) + 15.3403*X(condskim) - 74.8487*F(crm-rem) = 100*F(block)

5. Whey cream yield per batch

Whey cream yield per 100 pounds of input resources is computed using a formula described in Exhibit 3.1, and incorporated into the following equation computing the total amount of whey cream from a batch:

-7*F(crm-rem) + .5736*X(milk) + 7*X(crm-add) + .1556*X(nfdm) + .0576*X(condskim) = 100*F(wheycrm)

6. Separated whey yield per batch

Separated whey yield per 100 pounds of input resources is computed using a formula described in Exhibit 3.1, and incorporated into the following equation computing the total amount of separated whey from a batch:

-18.1513*F(crm-rem) + 89.1877*X(milk) + 18.2841*X(crm-add) + 51.7287*X(nfdm) + 84.6021*X(condskim) = 100*swy

where swy = amount of separated whey (pound).

7. Condensed whey (60% TS) yield per batch

Exhibit 3.2 describes in detail how the condensed whey yield per pound cheese is obtained. When total solid percent of separated whey is 6.5 percent, the condensed yield per pound cheese equals: F(condwhey) = .1083*swy.

8. Nonnegativity constraints of the ingredients and finished and by-products:

X(milk), X(crm-add), X(nfdm), X(condskim) $\geq$ 0

F(block), F(crm-rem), F(wheycrm), F(condwhey) $\geq$ 0.

A model minimizing total cheese manufacturing cost per pound cheese from a barrel Cheddar cheese vat

The objective function and constraints of the model which minimizes total cheese manufacturing cost per pound cheese from a barrel Cheddar cheese vat are determined without an explanation in detail since they are similar to those of the previous model.

Objective Function:

Minimize Z = COSTS/F(barrel)

where COSTS = Cost of manufacturing cheese and whey

Total cheese manufacturing costs per pound barrel Cheddar cheese are computed by dividing direct production costs by the amount of barrel cheese from a batch:

COSTS = .002*F(barrel) + .0016*F(crm-rem) + .0016*F(wheycrm)
+ .0178*F(condwhey) + .1197*X(milk) + .8235*X(crm-add)
+ .8100*X(nfdm) + .2390*X(condskim) + 369.9

where 369.90 = other direct production cost of processing 30000 pounds standardized milk.

Constraints:

1. Capacity of a cheese vat (batch size)

X(milk) + X(crm-add) + X(nfdm) + X(condskim) - F(crm-rem) = 30000

2. Acceptable range of casein to fat (C/F) ratios of cheese milk

Lower level of casein to fat ratio(C/F ≥ .68):

29.13*F(crm-rem) + .064*X(milk) - 29.21*X(crm-add) + 27.32*X(nfdm) + 8.9484*X(condskim) ≥ 0

Upper level of casein to fat ratio(C/F ≤ .70):

30.03*F(crm-rem) - .01*X(milk) - 30.11*X(crm-add) + 27.3*X(nfdm) + 8.941*X(condskim) ≤ 0

3. Maximum amount of cream that can be removed from raw milk

F(crm-rem) ≤ .0822*X(milk)

4. Cheese yield per batch

Cheese yields per 100 pounds of input resources are computed based on 38 percent of cheese moisture content.

100*F(barrel) = - 76.056*F(crm-rem) + 10.4039*X(milk) + 75.921*X(crm-add) + 48.8917*X(nfdm) + 15.876*X(condskim)

5. Whey cream yield per batch

-7*F(crm-rem) + .5736*X(milk) + 7*X(crm-add) + .1556*X(nfdm) + .0576*X(condskim) = 100*F(wheycrm)

6. Separated whey yield per batch

$$100*swy = -\ 17.079*F(crm\text{-}rem) + 89.0225*X(milk) + 17.079*X(crm\text{-}add) + 50.9527*X(nfdm) + 84.0664*X(condskim)$$

7. Condensed whey (60% TS) yield per batch

$$F(condwhey) = .1083*swy$$

8. Nonnegativity constraints of the ingredients and finished and by-products for Cheddar cheese

$$X(milk),\ X(crm\text{-}add),\ X(nfdm),\ X(condskim) \geq 0$$

$$F(block),\ F(crm\text{-}rem),\ F(wheycrm),\ F(condwhey) \geq 0$$

Results

The optimal formulations or recipes (per batch basis and per 100 pound basis) of block and barrel Cheddar cheeses are depicted in Tables 3.7, 3.8, 3.9 and 3.10. The optimal recipe for block cheese was determined at 0.68 of C/F ratio which is comparable to 54 percent of FDM level and 56 percent of MNFS level. The levels of FDM and MNFS are within the levels meeting the finest cheese quality (64). The optimal recipe for barrel cheese was determined at 0.68 of C/F ratio. This ratio is comparable to 54 percent of FDM level and 57 percent of MNFS level, which meet the levels of the first grade quality cheese.

Two representations of the models finding the most profitable formulations of block Cheddar cheese and the most cost efficient manufacture of barrel Cheddar cheese are shown in Exhibits 3.3 and 3.4, respectively. A mathematical programming software called GAMS (15) was used to solve the optimization problem. The representation of the models and their solution outputs by GAMS are presented in Appendix A.

The recipe based on the optimization do not necessarily produce quality cheese unless the amounts and processing conditions are controlled according to predefined processing guidelines. It is important to note that the optimal solution to the model may not be an optimal solution to the real situation. Variations in the quality attributes of ingredients such as milk, rennet, and starter culture may result in deviation from the guidelines of BOM. In order to accommodate the variations, BOM may need to be evolutionally adjusted. The optimal solution to the model can be altered by market conditions such as limited supplies or unavailability of input resources and a change in input resource costs, or manufacturing conditions. Since milk composition varies seasonally, standardization provides not only consistent cheese quality, but also a yardstick for profitability of cheese manufacture through the year. Limited supplies of a particular resource can be handled by adding the constraint regarding the amount of the input resource available.

A Bom Matrix for Cheddar Cheese Formulation

The Cheddar cheese formulation found through the optimization can be easily organized in a BOM matrix by using the direct relationship among finished products, input resources and by-products. Figure 3.2 shows per pound basis BOM matrix for the manufacture of Cheddar cheese. Table 3.11 describes the codings about the notation of products and input resources associated with the manufacture of Cheddar cheese and process cheese products. The standardized codings eliminate the possibility of using different names for the same item or using the same names for different items, and promotes the consistency and integration of data. Negative entry values in the BOM matrix indicate that whey cream and condensed whey are by-products resulting from the Cheddar cheese manufacture.

Gozinto Procedure Application to Process Cheese Manufacture

When a multi-staged process involving the production of the intermediate product is used to manufacture the finished product, building a BOM matrix is not simple. When the matrix theory is applied to a multi-staged food manufacturing facility, the Gozinto Procedure (GP) using matrix operations provides a structured way to define product recipe interactions by systematically arranging the recipes of multiple products. The GP can be used to show how the products would compete with one another for common resources in every stage of the manufacturing process.

The GP application to the process cheese manufacture is described as follows:

Step 1

Define the direct relationships between finished products and input resources through the formulations of the products. The formulations of process cheese food and spreads are shown in Tables 3.12 and 3.13, respectively. In the formulations, Cheddar cheese blends for process cheese food and spreads are intermediate products whose direct resources are young, medium-aged, and old-aged Cheddar cheeses. The young Cheddar cheese is a lower level intermediate product in the integrated production system because it is produced from the Cheddar cheese plant of the company. Condensed whey which is a by-product resulted from the Cheddar cheese manufacture is used as an input resource for the manufacture of process cheese spreads.

Step 2

Create a lower triangular, invertible recipe matrix **R** based on the relationships established in the step 1. The dimension of a lower triangular, square matrix **R** equals the number of items that would include finished products, direct single resources of finished products, intermediate products, direct single resources of intermediate products, and by-products. Direct single resources are defined as items which do not have any children items or direct resources. Each item is organized as an entry in **R** so that the input resources required by the item can be placed below the item in the column. The r_{ij} is an entry in the ith row and jth column of **R**, and represents the number of units of resource i required to produce a unit of parent item j. The unit of the item can be any unit form convenient to operation.

The recipe matrix **R** for the process cheese manufacture in Figure 3.3 organizes the requirements of direct input resources including labor and utilities per pound finished and intermediate products. The italic numbers in the figure indicate the levels of the products and input resources in the production:

1 : finished (unpackaged) products at the process cheese plant,

2 : intermediate products of the finished products,

3 : resources added to the process cheese cooker with Cheddar blends, *4* : resources of Cheddar blends.

Figure 3.4 presents an integrated recipe matrix $\mathbf{R}_I$ for process cheese products. $\mathbf{R}_I$ includes input resources and by-products of young barrel Cheddar cheese produced at the Cheddar cheese plant. This matrix organizes the entire flows of materials between the Cheddar and process cheese plants as well as inside the plants. The additional levels of input resources and by-products are:

5 : input resources of the young Cheddar cheese,

6 : by-products from the Cheddar cheese plant.

Negative entry values in the matrix indicate the amount of by-products resulted from a single unit of the parent item in the process. For example, '$\mathbf{R}_{(38,27)}$ = -.0557' means 0.557 pound of whey cream is produced as a by-product when one pound of young barrel Cheddar is manufactured.

Since the recipe matrix is built based on the direct relationships between products and their direct input resources, the matrix does not explicitly show the relationships between the products and their indirect resources. For example, **R** and $\mathbf{R}_I$ in Figures 3.3 and 3.4 do not provide the information about how much young barrel Cheddar or raw milk is needed to make a single unit (pound) of process cheese food. The total resource requirement of the product is useful to generate production planning information. The total resource requirement of the product is obtained by the matrix manipulation depicted in the next step.

Step 3

Create an identity matrix **I** of the same size as **R**. The inverse of the difference between **I** and **R** generates a total resource requirement matrix **T**: $\mathbf{T} = (\mathbf{I} - \mathbf{R})^{-1}$. The lower triangular matrix **T** with 1's on the principal diagonal has the same dimensions as **I** and **R**. **T** defines total resource requirements for each manufacturing stage. **T** describes which and how much input resources are required to make a unit of finished product or intermediate product, and how much by-products are resulted in for every stage of a manufacturing process. The $\mathbf{t}_{ij}$ is an entry in the ith row and jth column, which represents the amount of input resource i required to produce a unit of parent item j.

Step 4

As an extension to GP, a BOM matrix **B** is built by removing the columns having zero entries except 1's of diagonal entries as shown in Figure 3.5. These columns represent the resources which do not require any direct input resources. Thus, **B** retains the columns representing only the finished and intermediate products. The size of

BOM matrix in Figure 3.5 is reduced from 27x27 to 27x8, which enables computer users to more quickly store, retrieve and manage information.

The columns representing intermediate products provide the useful manufacturing information that serves the understanding of the product recipe structure including the level of resources and the differentiation of direct or indirect resources. The information is especially useful when the intermediate products are stored for bottleneck buffer, or sold for revenue sources without further processing. When the BOM is used for forecasting purpose or generating resource requirement, and the intermediate product is not sold, B may be furthermore compacted by keeping only the finished product columns. Appendix B shows how the GP is derived in the application to the process cheese manufacture.

Product Mix Optimization When Whole Batching Is Involved

Many food products, such as cheese, ice cream, canned vegetable and processed meat, are manufactured through one or more batch process. A batch process occurs when a predefined quantity of a formula is prepared according to a specification in a single operation. Producing batches is part of a manufacturing sequence for intermediate or finished products in a multi-staged process. As illustrated in the case of natural cheese vats and process cheese cookers, the batch output from a single batch type may be directly or indirectly used to produce several finished products, or several batch types may be used sequentially or simultaneously to produce a finished product.

Continuous cheese manufacturing system implies a continuous flow of milk and curd through the entire cheese manufacturing system (85). Automated equipment for the transfer of milk or whey, heat treatment, temperature control, CIP cleaning, starter by injection, curd stirring, whey drainage, curd milling, curd salting, mould filling, cheese pressing, and movement of cheese into and out of storage room is now available. While most processes of the manufacturing system use continuous processing equipment, there are few continuous cheese manufacturing systems in commercial use due to technological or economic reasons. The continuous process is not appropriate for supporting time-demanding blending necessary to promote desired quality attributes of cheese products. For example, ingredients are mixed and heated in a cheese vat for a specified time, even though subsequent processes operate on a continuous basis. A process cheese cooker is also a batchwise system, where processing is done on a batch basis through a cooking operation but a sufficient number of cookers are used to provide a continuous flow of cheese to subsequent packaging operations. Such manufacturing systems can be defined as a semi-continuous or batchwise-continuous system in a strict manner. The batchwise-continuous or semi-continuous system is expected to be dominant for some years in harder types of cheese such as Cheddar.

When the batch process is involved in a manufacturing system, determining a product mix associated with a number of constraints is not a simple matter. When a product is constantly demanded and storable with low inventory costs, whole batch production is preferred for managerial and technical conveniences. The whole batching policy is a common practice in the natural and process cheese plants because most of natural cheeses are ripened for a certain period and process cheeses are usually storable up to 3 months. It is complex to optimize product mix decisions under the whole batching policy due to the restriction of batch units to integer values. The problem becomes more complicated when several batch types are involved or when multiple products are produced entirely or partly from the same batch type. The constraints restricting the plant capacity and raw material supplies add to the complexity of the optimization problem. Production plans adjusted to whole batching policy may result in potential shortages or excess of products. It does not, however, mean that allowing partial batches is preferable. Even though partial batching may be economically desirable, it may generate variable yields or variable quality attributes. In general, whole batching is preferred for the products with low perishability, mass production, or sufficient and constant demand, while partial batching is used for highly perishable products with an intermittent demand, products requiring expensive materials like seafood and nutrasweet, or Just-in-Time production (JIT) in the food industry.

A Mathematical Model for Product Mix Optimization

A problem for optimizing the product mix of Cheddar cheese, process cheese products and by-products is formulated as a mixed integer programming (MIP) model. The product mix optimization model is built by fixing the size of each batch type, and allowing the number of batch units for each batch type variable within a capacity and under a integrality condition. The objective of the model is to find the most profitable product mix under the whole batching restriction. To measure the economic consequences of product mix and batching decisions, the solution of the MIP model will be compared to the solution when the partial batching is allowed (i.e., linear programming model solution). In the model capital letters indicate variables, while lower case letters constants, and underlined lower case letters vectors. The MIP model is described as follows:

MAXIMIZE $f(\underline{x},\underline{w},\underline{y})$ (3-1)

$$f(\underline{x},\underline{w},\underline{y}) = \sum_{i \in I} \sum_{j \in J} p_{ij}X_{ij} + \sum_{k \in K} s_k W_k + \sum_{h \in H} c_h Y_h - c_1 \sum_{i \in I} \sum_{j \in J} b_{ij}X_{ij}$$

SUBJECT TO

$$Ze_{ij} + (1-Z)d_{ij} \le X_{ij} \le Zd_{ij} + (1-Z)u_{ij}, \ i \in I, \ j \in J \quad (3\text{-}2)$$

$$\sum_{j \in J} X_{ij} = \sigma B_i, \ i \in I \quad (3\text{-}3)$$

$$\sum_{i \in I} B_i \le b \quad (3\text{-}4)$$

$$\sum_{i \in I} \sum_{j \in J} a_{ij}X_{ij} \le W_2 \quad (3\text{-}5)$$

$$W_k = t_k V_k, \ k \in K \quad (3\text{-}6)$$

$$\sum_{k \in K} V_k \le v \quad (3\text{-}7)$$

$$Y_h = \sum_{k \in K} r_{kh} W_k, \ h \in H \quad (3\text{-}8)$$

$$Z = 0 \text{ if } \sum_{i \in I} \left\| \sum_{j \in J} \frac{1}{\sigma} d_{ij} \right\| \le 750, \ Z = 1 \text{ otherwise.} \quad (3\text{-}9)$$

$$X_{ij} \text{ integer, } i \in I \text{ and } j \in J \quad (3\text{-}10)$$

$$B_i \text{ integer, } i \in I \quad (3\text{-}11)$$

$$V_k \text{ integer, } k \in K \quad (3\text{-}12)$$

where:

I = the index set of process cheese product types with I = {1, 2, - - -, 6}

i = 1: process cheese food,

2: plain (Cheddar) process cheese spread,

3: chive & onion process cheese spread,

4: nacho & red pepper process cheese spread,

5: bacon & hickory smoke process cheese spread,

6: salami & hickory smoke process cheese spread;

J = the index set of process cheese product package options with J = {1, 2}

j = 1: a case of 50, 8 oz. cups(50/8),

2: a case of 25, 16 oz. cups(25/16);

K = the index set of Cheddar cheese product types or batch types with K = {1, 2}

k = 1: block Cheddar cheese,

2: barrel Cheddar cheese;

H = the index set of by-products from Cheddar cheese plant with H = {1, 2}

h = 1: condensed whey,

2: whey cream;

X_{ij} = number of cases of a process cheese product type i with a package option j (product ij) in a production target;

W_k = amount of Cheddar cheese product type k in a production target;

Y_h = amount of by-product h from the Cheddar cheese manufacture;

B_i = number of batches (process cheese cookers) for process cheese product type i;

V_k = number of batches (cheese vats) for Cheddar cheese type k;

Z = an integer variable to handle whole batch production within a manufacturing capacity

Z = 1 when the projected demand for products is not more than the manufacturing capacity (number of batches),

Z = 0 otherwise;

p_{ij} = profit contribution margin per case of process cheese product ij (refer to Table 3.14);

s_k = profit contribution margin per pound Cheddar cheese type k, $s_1 = .011$, $s_2 = .0$;

c_h = profit contribution margin per pound by-product h, $c_1 = .0602$, $c_2 = .7784$;

d_{ij} = projected demand of process cheese product ij for a specific time period (e.g. month);

e_{ij} = production that must at least be achieved for product ij;

u_{ij} = upper production level that may be allowed for product ij;

$\|\theta\|$ = the smallest integer not less than θ;

a_{ij} = young barrel Cheddar requirement (pounds) per case of process cheese product ij (refer to Table 3.16);

b_{ij} = condensed whey requirement (pounds) per case of process cheese product ij (refer to Table 3.16);

v = Cheddar cheese production capacity in terms of the number of cheese vats, v = 720 (monthly capacity);

b = process cheese production capacity in terms of the number of cheese cookers, b = 750 (monthly capacity);

t_k = amount of Cheddar cheese product type k production per cheese vat, $t_1 = 3{,}113.90$, $t_2 = 3{,}164.14$;

r_{kh} = yield of by-product h per pound Cheddar cheese product type k,

$r_{11} = .9292$, $r_{12} = .9125$, $r_{21} = .0566$, $r_{22} = .0557$;

σ = number of cases produced from a whole process cheese cooker,

$$\sigma = \frac{2000}{25} = 80$$

where 2,000 = a unit batch size (pounds) of process cheese products,

25 = pounds of process cheese product in a case, regardless of product types or package options.

Assumptions

Supplies of input resources except raw milk are assumed limitless. If the supply of a particular resource is restrained for any reason, the production of the products using the resource will be restricted by placing a constraint limiting the total usage of the resource. The resource requirement of the products is obtained from the BOM matrix. Thirty-day warehousing capacity of Cheddar cheese and 50-day warehousing capacity of process cheese products are assumed enough not to put the constraints for inventories. The constraints for inventories can be put when needed by converting the unit of the warehousing capacity into the unit of the products. The whole batching policy at the Cheddar cheese and process cheese plants is assumed because the cheese products are storable for a relatively long time, and are constantly demanded. Unit prices and direct production costs of Cheddar cheese products are described in Tables 3.2 and 3.10. Table 3.14 lists direct production costs and profit contribution margins per case of the process cheese products.

Decision variables

Decision variables for the product mix optimization are identified with process cheese products which may be produced at the process cheese plant, and Cheddar cheese products and by-products which may be produced at the Cheddar cheese plant. Decision variables and their notations are listed in Table 3.15.

Constraints

(3-2,3-9): These constraints ensure an acceptable range of production target for each product. An objectively driven range of production target offers a slack for the whole batching policy by providing a flexibility to produce an integer number of batch units. The range is determined according to the projected demand for products. If the projected demand for the products does not exceed the manufacturing capacity (number of batch units), a lower level of the range is the projected demand of the products and an upper level is the maximum production that is allowable for the products. Otherwise, the lower level is the minimum production that must be achieved for the products and the upper level is the projected demand of the products.

(3-3): This constraint indicates the whole batching policy for each process cheese product batch type by ensuring the sum of the case production of the same product type equals the batch production of the product type.

(3-4): This constraint shows the monthly production capacity that limits the total batch production of process cheese products.

(3-5): This ensures the production of young barrel Cheddar cheese to meet the demand of process cheese products.

(3-6): This constraint indicates the amount of Cheddar cheese type manufactured at the Cheddar cheese plant.

(3-7): This ensures the monthly production capacity limiting the total batch production of Cheddar cheese.

(3-8): This indicates the total amount of by-products produced at the Cheddar cheese plant.

(3-10): This constraint ensures the policy that allows only whole case production (50/8 or 25/16).

(3-11): This constraint assures a whole batching policy for process cheese cookers at the process cheese plant.

(3-12): A whole batching policy for cheese vats at the Cheddar cheese plant is ensured.

Objective function

To find a satisfactory product mix is one of the objectives for the production planning framework. The problem of optimizing the cheese production takes into account direct production costs that include costs of raw materials, packaging materials, labor, utilities, and production supplies (88). The cheese products have different costs and selling prices, and accordingly different profit contribution margins. In optimizing the product mix, profit maximization would be an appropriate definition for the objective function since the minimization of the manufacturing cost does not consider the selling price or profit contribution margin and accordingly may not provide a satisfactory product mix. The objective function of the model is defined as the maximization of total profit contribution margins from the projected sales of the cheese products and by-products under the constraints of the plant capacity and whole batching restriction. The profit contribution margin per unit of a product is determined by selling price minus direct production cost per unit of the product.

Condensed whey which is a by-product of the Cheddar cheese manufacture is used as an input resource for process cheese products. If the condensed whey production is more than the requirements of process cheese products then the extra condensed whey is supposed to be sold at the market price. If the condensed whey production is less than the demand of process cheese products then the amount of condensed whey necessary for the process cheese manufacture will be purchased in the market at the market price. On the basis of this assumption, the objective function is written in each situation as follows:

If the condensed whey production is not less than the requirements of process cheese products, the objective function is:

$$f(\underline{X},\underline{W},\underline{Y})=\sum_{i\epsilon I}\sum_{j\epsilon J} p_{ij}X_{ij} + \sum_{k\epsilon K} s_k W_k + (\alpha-\beta)(Y_1 - \sum_{i\epsilon I}\sum_{j\epsilon J} b_{ij}X_{ij}) + \tau Y_2$$

Otherwise, the objective function is:

$$f(\underline{X},\underline{W},\underline{Y})=\sum_{i\epsilon I}\sum_{j\epsilon J} p_{ij}X_{ij} + \sum_{k\epsilon K} s_k W_k + \alpha(Y_1 - \sum_{i\epsilon I}\sum_{j\epsilon J} b_{ij}X_{ij}) + \tau Y_2$$

where

α = market price per pound condensed whey

β = production cost per pound condensed whey

τ = profit contribution margin per pound whey cream which is a by-product from Cheddar cheese manufacture

$Y_1-\sum_{i\epsilon I}\sum_{j\epsilon J} b_{ij}X_{ij}$ = amount of condensed whey available for sales or amount of condensed whey needed to purchase.

These two different objective functions can be combined into an objective function by implementing a zero-one variable N as follows:

$$f(\underline{x},\underline{w},\underline{y})=\sum_{i\epsilon I}\sum_{j\epsilon J} p_{ij}X_{ij} + \sum_{k\epsilon K} s_kW_k + \tau Y_2 +$$

$$\{N(\alpha - \beta) + (1 - N)\alpha\}(Y_1 - \sum_{i\epsilon I}\sum_{j\epsilon J} b_{ij}X_{ij}),$$

$$N = \begin{cases} 1 \text{ if } Y_1 \geq \sum_{i\epsilon I}\sum_{j\epsilon J} b_{ij}X_{ij}, \\ 0 \text{ otherwise.} \end{cases}$$

It is likely that the high yield of condensed whey in the Cheddar cheese manufacture makes the condensed whey production enough to meet the demand of the process cheese plant. The integrated BOM matrix indicates a pound of process cheese products requires less than one third amount of the condensed whey produced from the required amount of young barrel Cheddar. Considering the condensed whey production from the block Cheddar production which is almost three times greater than the barrel Cheddar production, there will certainly be a sufficient amount of the condensed whey to satisfy the demand of process cheese products. When the Cheddar cheese plant operates at a full capacity, the plant produces approximately 2,080,800 pounds of condensed whey per month. When the process cheese plant operates at full capacities, the process cheese plant demands less than 90,000 pounds of condensed whey, which is only 4.3 percent of total condensed whey production in the Cheddar cheese plant with the full capacity. Unless the Cheddar cheese plant operates at less than a 4.3 percent level, condensed whey would not be purchased. Actually, the 4.3 percent operating level is unrealistic even in the condition of very short milk supplies. Thus, the optimization model to maximize total profit contributions is made under the assumption that there will be enough condensed whey production to meet the requirement of process cheese products. Considering this realistic situation, the objective function is represented as written in the model:

$$f(\underline{x},\underline{w},\underline{y}) = \sum_{i\epsilon I}\sum_{j\epsilon J} p_{ij}X_{ij} + \sum_{k\epsilon K} s_k W_k + (\alpha-\beta)(Y_1 - \sum_{i\epsilon I}\sum_{j\epsilon J} b_{ij}X_{ij}) + \tau Y_2$$

$$= \sum_{i\epsilon I}\sum_{j\epsilon J} p_{ij}X_{ij} + \sum_{k\epsilon K} s_k W_k + \sum_{h\epsilon H} c_h Y_h - c_1 \sum_{i\epsilon I}\sum_{j\epsilon J} b_{ij}X_{ij}$$

where

c_h = profit contribution margin per pound by-product h from Cheddar cheese manufacturing

$c_1 = \alpha - \beta$

$c_2 = \tau$

$h \epsilon H,\ H = \{1,2\}$.

This objective function is specifically expressed below.

$f(\underline{x},\underline{w},\underline{y})$ =

Profit contribution from process cheese products sales(P1)

+ Profit contribution from Cheddar cheese products sales(P2)

+ Profit contribution from condensed whey sales(P3)

+ Profit contribution from whey cream sales(P4)

$$P1 = 6.5480X_{11} + 7.0480X_{12} + 7.1130X_{21} + 7.6130X_{22} + 4.4730X_{31} + 4.9730X_{32} + 4.5380X_{41} + 5.0380X_{42} + 4.5755X_{51} + 5.0755X_{52} + 3.7630X_{61} + 4.2630X_{62}$$

$$P2 = .011W_1 + .0W_2$$

$$P3 = .0602(Y_1 - E)$$

$$P4 = .7784Y_2$$

$$E = .0X_{11} + .0X_{12} + 2.50X_{21} + 2.50X_{22} + 2.375X_{31} + 2.375X_{32} + 2.175X_{41} + 2.175X_{42} + 2.30X_{51} + 2.30X_{52} + 2.325X_{61} + 2.325X_{62}$$

where

E = Condensed whey requirement(pounds) per case of process cheese product.

To illustrate the use of the IP model, monthly demands for process cheese products were assumed as shown in Table 3.17. An IP model is specifically expressed using the demand of October as follows:

<u>An IP Model for Product Mix Optimization</u>

MAXIMIZE $f(\underline{x},\underline{w},\underline{y})$ =

$$6.5480X_{11} + 7.0480X_{12} + 6.9625X_{21} + 7.4625X_{22} + 4.3391X_{31} + 4.8391X_{32} + 4.4176X_{41} + 4.9176X_{42} + 4.4400X_{51} + 4.9400X_{52} + 3.6275X_{61} + 4.1275X_{62} + .011W_1 + .0602Y_1 + .7784Y_2$$

SUBJECT TO

$$13950 \le X_{11} \le 14229, \quad 7000 \le X_{12} \le 7140$$

$$5400 \le X_{21} \le 5508, \quad 2960 \le X_{22} \le 3019$$

$$5100 \le X_{31} \le 5254, \quad 1990 \le X_{32} \le 2030$$

$$5500 \le X_{41} \le 5610, \quad 3300 \le X_{42} \le 3366$$

$$4100 \le X_{51} \le 4182, \quad 2400 \le X_{52} \le 2448$$

$$4800 \le X_{61} \le 4896, \quad 2950 \le X_{62} \le 3009$$

$$X_{11} + X_{12} = 80B_1, \quad X_{21} + X_{22} = 80B_2$$

$$X_{31} + X_{32} = 80B_3, \quad X_{41} + X_{42} = 80B_4$$

$$X_{51} + X_{52} = 80B_5, \quad X_{61} + X_{62} = 80B_6$$

$$B_1 + B_2 + B_3 + B_4 + B_5 + B_6 \le 750$$

$$10.5X_{11} + 10.5X_{12} + 11.55X_{21} + 11.55X_{22} + 10.5X_{31} + 10.5X_{32} + 10.325X_{41} + 10.325X_{42} + 10.5X_{51} + 10.5X_{52} + 10.5X_{61} + 10.5X_{62} \le W_2$$

$$W_1 - 3113.90V_1 = 0, \quad W_2 - 3164.14V_2 = 0$$

$$V_1 + V_2 \le 720$$

$$Y_1 = .9292W_1 + .9125W_2, \quad Y_2 = .0566W_1 + .0557W_2$$

$$Y_1 - 2.5X_{21} - 2.5X_{22} - 2.25X_{31} - 2.25X_{32} - 2.0X_{41} - 2.0X_{42} - 2.25X_{51} - 2.25X_{52} - 2.25X_{61} - 2.25X_{62} = WS$$

X_{ij} integer, $i \in I$ and $j \in J$

B_i integer, $i \in I$

V_k integer, $k \in K$.

In many IP problems, all decision variables are binary (0/1) variables like facility location and traveling salesman problems. In most general application problems, however, some or all of the decision variables have general integer values requiring more than two possible values. Even in this case, general integer variables can be replaced with a binary representation to modify the problem to a binary IP. Suppose the bounds on an integer variable X are:

$$0 \leq X \leq u, \quad \text{where } 2^{k-1} \leq u \leq 2^k,$$

then each feasible value of X can be uniquely expressed as

$$X = \sum_{i=0}^{k} 2^i Y_i, \quad \text{where } Y_i = (0,1),\ i = 0,1, ---, k.$$

By substituting for X in terms of Y_i, general integer problems become a mixed or pure zero-one problem. This substitution may be reasonable when the number of binary variables is not large. A commercial package LINDO (The Scientific Press, Copyright 1984) requires converting the general IP into the binary IP. If a variable in the IP problem can take on any value within a specific range but IP code with zero/one capability is only available like LINDO, the general integer variables should be transformed into zero-one variables.

Results and Discussion

The result of the IP problem is shown in Table 3.18 with a LP solution that relaxed the integrality condition. The objective solution of the IP is less than that of the LP by $ 216.5, but the IP solution clearly shows whole batches for all batch types. Results of November and December production optimization problems in Tables 3.19 and 3.20 also indicate whole batches for all batch types but the differences between LP and IP solutions are bigger as $317.3 and $1212.1, respectively. The results of the tables indicate that the production plans in October and December require full production capacity, while in November the production falls below full capacity.

The IP problem sometimes may be solved mainly due to many integrality restrictions. This difficulty can be solved by allowing partial batches to some batch types. It may be also possible to solve the problem as a continuous model by simply applying the simplex algorithm, and then round the continuous optimal solution to a feasible integer solution. In the latter case, it would be necessary to reformulate the original problem with the rounded numbers and solve the problem in order to determine the final product mix based on the newly rounded batch mix. But it is not guaranteed that the rounded solution would satisfy capacity or demand constraints. When the real number of process cheese batch cooker for each process cheese product type is rounded to the nearest integer in LP solution, the sum of the rounded integer numbers can be more than the maximum number of cookers available or the demand for a specific product may not be satisfied.

In the LP solution, the sum of the rounded integer numbers for process cheese cookers exceeds 720 by 2. This rounding problem can also occur even when partial batches are allowed because a partial batch requires a single cooker as a whole batch does. Even though the real numbers of cookers are rounded within the capacity, another integrality restriction to the number of Cheddar cheese vats does not ensure that the rounded integer number of cheese vats can meet the requirements of process cheese batches whose numbers are rounded to integer numbers.

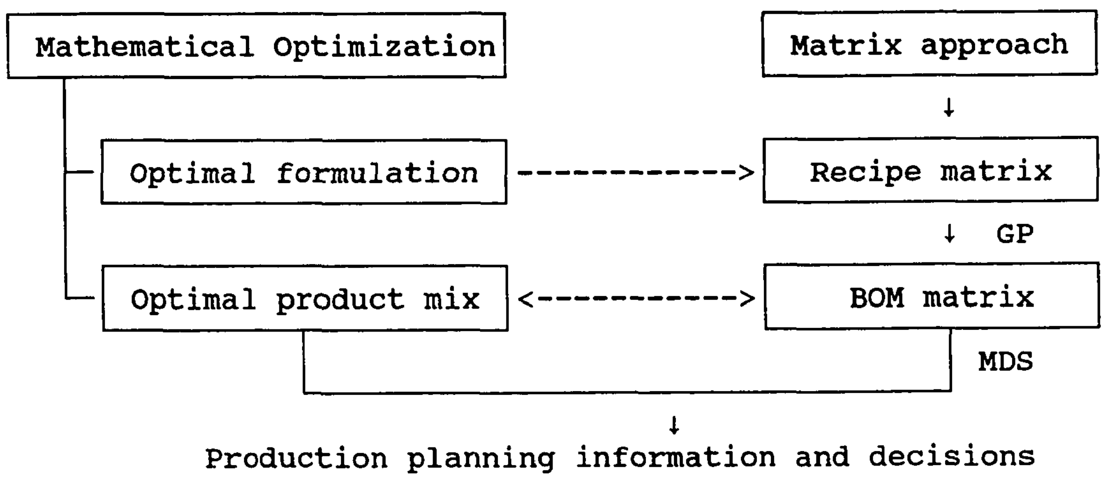


Figure 3.1. A scheme of generating planning information in the production planning framework

Table 3.1. Input resource information available for use in the model optimizing Cheddar cheese formulation[a]

Resources	Unit of Measure	Amount Available	Fat%	Casein%	Cost($) per Unit
Raw milk	pound	720,000[a]	3.70	2.58	.1197[b]
Non-Fat Dry Milk(NFDM)	pound	as needed	1.00	28.00	.8100
Cream	pound	as needed	45.00	1.39	.8235[c]
Condensed skim milk	pound	as needed	0.37	9.20	.2390
Rennet	ounce	as needed	--	--	.4613
Starter cultures	pound	as needed	--	--	.4432
Color	ounce	as needed	--	--	.0620
Salt	pound	as needed	--	--	.1200
Direct labor	hour	as needed	--	--	10.0000
Electricity	KWH	as needed	--	--	.0650
Natural gas	therm	as needed	--	--	.4500
Water	gallon	as needed	--	--	.0000
Sewage	gallon	as needed	--	--	.0012
Packaging material Block Cheese	unit[d]	as needed	--	--	.6000
Barrel Cheese	Unit[e]	as needed	--	--	1.0000

[a] Amount indicates daily processing capacity based on the full capacity utilization rate.

[b] Base milk price(3.5% fat,3.2 % protein) is $11.68 per 100 lbs. Milk price was computed based on $1.45 per pound fat and $.12 per .1 point above 3.2% protein. The 3.7% fat milk contains 3.2% protein.

[c] Cream price was computed based on $1.83 per pound fat.

[d] A unit equals the packaging material requirement for one unit of block Cheddar cheese. Per lb cost is $.015.

[e] A unit equals the packaging material requirement for one unit of barrel Cheddar cheese. Per lb cost is $.002.

Table 3.2. Finished products and by-products available at the Cheddar cheese plant

Product	Unit of Measure	Fat %	Moisture %	Price($) per Unit
40-lb Block	pound	(1)	37	1.3075
500-lb Barrel	pound	(1)	38	NM[a]
Cream removed	pound	45	55	.8235[b]
Whey Cream	pound	45	55	.7875[c]
Condensed whey	pound	-	40	.0780[d]

NM: not meaningful
(1): determined after the model is solved.

[a] Barrel Cheddar is assumed to be used at the process cheese plant.

[b] Unit price of cream removed was computed based on $1.83 per pound fat.

[c] Unit price of whey cream was computed based on $1.75 per pound fat.

[d] Unit price of condensed whey was computed based on $.13 per pound solid.

[bcd] Product unit prices are only applied when the products are sold.

Table 3.3. Constraints in the model optimizing Cheddar cheese formulation

Item	Range
Fat in the Dry Matter(FDM)	53.26 - 53.90 percent
Casein and Fat Ratio	.68 - .70
Fat Retention	.93
Casein Retention	.96
Salt Factor	1.09
Moisture(%) of	
Block Cheddar Cheese	37.00 percent
Barrel Cheddar Cheese	38.00 percent
Moisture(%) in the Non-Fat Substance(MNFS)	55.68 - 56.00 percent[a]
Capacity of a cooking vat	30,000 lbs of standardized milk

[a] $MNFS = \frac{M}{100 - F}$ and $FDM = \frac{F}{100 - M}$

$F = (100 - M)FDM$

Therefore, $MNFS = \frac{M}{100 - (100 - M)FDM}$

The range of MNFS is $55.68\% \le MNFS \le 56.00\%$, because $M = 37\%$ and $53.26\% \le FDM \le 53.90\%$.

where:
M = Moisture content(%) of cheese,
F = Fat content(%) of cheese,
MNFS = Moisture (%) in the Non-Fat Substance of cheese,
FDM = Fat in the Dry Matter of cheese.

Table 3.4. Decision variables in the GAMS model optimizing Cheddar cheese formulation

Item	Decision variable[a]
Output products	
Block Cheddar cheese	F(block)
Barrel Cheddar cheese	F(barrel)
Cream removed	F(crm-rem)
Whey cream	F(wheycrm)
Condensed whey	F(condwhey)
Ingredients	
Raw milk	X(milk)
Cream added to cheese milk	X(crm-add)
Non Fat Dry Milk(NFDM)	X(nfdm)
Condensed skim milk	X(condskim)

[a] The amounts of starter culture, rennet, color(annatto), salt, labor and utilities used for a batch (cheese vat) are considered constant, regardless of milk standardization. Accordingly, per batch costs of these resources are fixed.

Table 3.5. Comparison of optimal solutions of different objective function measures for the block Cheddar model[a]

Objective Measure :	Maximize Profits	Maximize Profit margin[b]	Minimize Costs	Minimize Cost/lb cheese[c]
Objective value	$ 321.943	7.26%	$ 4067.229	$ 1.321
Profits($)	321.943	321.943	310.435	321.943
Profit margin(%)	7.26	7.26	7.09	7.26
Revenue($)	4435.883	4435.883	4377.664	4435.883
Costs($)	4113.940	4113.940	4067.229	4113.940
Cost/lb cheese($)	1.321	1.321	1.324	1.321
Production of output products				
Cheese(lb)	3113.899	3113.899	3071.610	3113.899
Cream removed(lb)	-	-	-	-
Whey cream(lb)	176.295	176.295	172.080	176.295
Separated whey(lb)	26709.806	26709.806	26756.310	26709.806
Condensed whey(lb)	2892.672	2892.672	2897.708	2892.672
Ingredients used for milk standardization				
Raw milk(lb)	29934.413	29934.413	30000.000	29934.413
Cream(lb)	65.587	65.587	-	65.587
NFDM(lb)	-	-	-	-
Cond. skim milk(lb)	-	-	-	-
Yields				
Cheese yield(%)	10.380	10.380	10.240	10.380
Whey cream yield per lb cheese(lb)	.057	.057	.056	.057
Separated whey yield per lb cheese(lb)	8.577	8.577	8.711	8.577
Condensed whey yield per lb cheese(lb)	.929	.929	.943	.929

[a] Large volume purchasing discounts may differentiate the results among the different objective measures.
[b] Profit margin(%) = Profit/Revenue * 100
[c] Cost includes whey cream processing and condensing whey costs.

Table 3.6. Comparison of optimal solutions of different objective function measures for the barrel Cheddar model

Objective Measure :	Minimize Costs	Minimize Cost/lb cheese[a]
Objective value	$ 4018.901	$ 1.285
Costs($)	4018.901	4065.063
Cost/lb cheese($)	1.288	1.285
Production of output products		
Cheese(lb)	3121.170	3164.141
Cream removed(lb)	-	-
Whey cream(lb)	172.080	176.295
Separated whey(lb)	26706.750	26659.564
Condensed whey(lb)	2892.341	2887.231
Ingredients used for milk standardization		
Raw milk(lb)	30000.000	29934.413
Cream(lb)	-	65.587
NFDM(lb)	-	-
Cond. skim milk(lb)	-	-
Yields		
Cheese yield(%)	10.400	10.550
Whey cream yield per lb cheese(lb)	.055	.056
Separated whey yiel per lb cheese(lb)	8.557	8.426
Condensed whey yiel per lb cheese(lb)	.927	.913

[a] Cost includes whey cream processing & condensing whey costs.

Table 3.7. Optimal formulation and direct production costs of block Cheddar cheese (per batch basis)[a]

Resource	Unit of Measure	Quantity per Vat	Cost($)[b]
Ingredients			
Raw Milk	pound	29,934.41	3,583.15
Cream added	pound	65.59	54.01
(Total Cost for milk ingredients)			(3,637.16)
Rennet	ounce	90.0	39.90
Starter cultures[c]	pound	210.0	96.90
Color	ounce	30.0	1.80
Salt	pound	45.0	5.40
(Total cost for other ingredients)			(144.00)
Packaging material	unit	77.85	46.71
Direct labor	hour	17.70	177.00
Utilities			
Electricity	KWH	332.31	21.60
Natural Gas	therm	42.67	19.20
Water	gallon	360.0	-
Sewage	gallon	2,499.90	3.00
(Total Utility cost)			(43.80)
Production supplies	NM	NM	3.00
Maintenance	NM	NM	10.50
Whey cream process	pound	176.29	.28
Condensing whey	pound	2892.67	51.49
TOTAL COSTS			**4,113.94**

NM = Not Meaningful

[a] The vat capacity for standardized milk is 30,000 lbs, which excludes the amount of rennet, culture, color and salt.

[b] This cost indicates direct production costs associated with cheese resulting from one vat.

[c] Age 15 hours, Acidity .7%

Table 3.8. Optimal formulation and direct production costs of barrel Cheddar cheese (per batch basis)[a]

Resource	Unit of Measure	Quantity per Vat	Cost($)[b]
Ingredients			
Raw Milk	pound	29,934.41	3,583.15
Cream added	pound	65.59	54.01
(Total Cost for milk ingredients)			(3,637.16)
Rennet	ounce	90.00	39.90
Starter culture[c]	pound	210.00	96.90
Color	ounce	30.00	1.80
Salt	pound	45.00	5.40
(Total cost for other ingredients)			(144.00)
Packaging material	unit	6.33	6.33
Direct labor	hour	17.70	177.00
Utilities			
Electricity	KWH	203.08	13.20
Natural Gas	therm	42.67	19.20
Water	gallon	360.00	-
Sewage	gallon	2,499.90	3.00
(Total Utility cost)			(35.40)
Production supplies	NM	NM	3.00
Maintenance	NM	NM	10.50
Whey cream process	pound	176.29	.28
Condensing whey	pound	2887.23	51.39
TOTAL COSTS			**4,065.06**

NM = Not Meaningful

[a] The vat capacity for standardized milk is 30,000 lbs, which excludes the amount of rennet, culture, color and salt.

[b] This cost indicates direct production costs associated with cheese resulting from one vat.

[c] Age 15 hours, Acidity .7%

Table 3.9. Optimal formulation of block and barrel Cheddar cheese (per 100 lb basis)[a]

Resources	Unit of Measure	Block Cheddar	Barrel Cheddar
Ingredients			
Raw Milk	pound	961.32	946.05
Cream added	pound	2.10	2.07
Rennet	ounce	2.89	2.84
Starter culture	pound	6.74	6.64
Color	ounce	0.96	0.95
Salt	pound	1.45	1.42
Packaging material	unit	2.50	0.20
Direct labor	hour	0.57	0.57
Utilities			
Electricity	KWH	10.67	6.42
Natural Gas	therm	1.37	1.35
Water	gallon	1.16	0.38
Sewage	gallon	80.28	79.01
Production supplies	NM	NM	NM
Maintenance	NM	NM	NM
Whey cream process	pound	5.66	5.57
Condensing whey	pound	92.89	91.25

NM = Not Meaningful

[a] The quantity per 100 lb cheese is computed based on the cheese yield of 3113.899 lb and 3164.141 lb per vat of block and barrel Cheddar cheeses, respectively.

Table 3.10. Direct production costs and by-product yields per 100 lbs block and barrel Cheddar cheese

Cost items\cheese	Block	Barrel
Direct Production Cost($) per 100 lbs		
Ingredient cost	$121.36	$119.50
Packaging cost	1.50	.20[a]
Direct labor & utility cost	7.11	6.72
Cost of production supplies and maintenance	.45	.42
Cost of removing whey cream	.01	.01
Cost of condensing whey	1.65	1.62
Total costs($)	132.08	128.47
By-product Yield(lb) per 100 lbs		
Whey cream	5.66	5.57
Condensed whey	92.90	91.25

[a] Recycling of 500-lb drum between process and Cheddar cheese plants reduces the cost.

Exhibit 3.1. Computation of various yields associated with cheese manufacture

---

$$CY_j = \frac{[FR*F_j + CR*C_j]*SR}{1 - W} \tag{1}$$

$$WCY_j = \frac{[F_j - FR*F_j]*WFR}{WF} = \frac{(1 - FR)*F_j*WFR}{WF} \tag{2}$$

$$SWY_j = 100 - CY_j - WCY_j \tag{3}$$

where

CY_j = cheese yield per 100 lbs input resource j,

WCY_j = whey cream yield per 100 lbs input resource j,

SWY_j = separated whey yield per 100 lbs input resource j,

FR = fat retention percentage divided by 100,

F_j = fat percentage of an input resource j,

CR = casein retention percentage devided by 100,

C_j = casein percentage of an input resource j,

SR = salt solids retention factor,

W = cheese moisture percentage divided by 100,

WFR = whey fat recovery percentage divided by 100,

WF = fat percentage of whey cream divided by 100.

(Exhibit 3.1. continued)

$$TC = \sum_{j=1}^{n} CY_j X_j \,/\, 100 \tag{4}$$

$$TWC = \sum_{j=1}^{n} WCY_j X_j \,/\, 100 \tag{5}$$

$$TSW = \sum_{j=1}^{n} SWY_j X_j \,/\, 100 \tag{6}$$

$$CY(\%) = (TC/\sum_{j=1}^{n} X_j)*100 \tag{7}$$

$$WCY = TWC/TC \tag{8}$$

$$SWY = TSW/TC \tag{9}$$

where

TC = total cheese amount resulting from the use of input resources,

TWC = total whey cream amount from the use of input resources,

TSW = total separated whey amount from the use of input resources,

X_j = amount of an input resource j that has a positive cheese yield,

n = number of input resources that produce positive cheese yield,

CY = cheese yield per lb input resources (%),

WCY = whey cream yield per lb cheese,

SWY = separated whey yield per lb cheese.

Exhibit 3.2. Computation of condensed whey yield and cost

Separated whey(6.5% TS) yield per pound cheese = SWY = α lbs

Amount of cheddar cheese produced = 10000 lbs

Then, amount of Separated whey = 10000α lbs

Amount of total solids(TS) from the separated whey
= .065(10000α) = 650α lbs
= Amount of total solids in condensed whey(60% TS)

Amount of the condensed whey = $\frac{650\alpha(100)}{60}$ = 1083.3333α lbs

Thus, condensed whey yield per lb cheese = .1083α lbs

Evaporator operation time spent to produce condensed whey = t

Feed amount of the separated whey during t = 10000α lbs

Amount of the condensed whey produced during t = 1083.3333α lbs

Evaporation amount during t= 10000α-1083.3333α = 8916.6667α lbs

Efficiency ratio of a triple effect evaporator = 3:1

Thus, steam amount required = $\frac{8916.6667\alpha}{3}$ = 2972.2222α lbs

Steam cost = \$6.50 per 1000 lbs

Evaporation(steam) cost = $\frac{2972.2222\alpha(6.50)}{1000}$ = \$19.3190α

Revenue from the condensed whey = .078*1083.3333α = \$84.5α

Profit from the operation of condensing whey = \$65.1850α

Thus, profit per pound condensed whey = $\frac{65.1850\alpha}{1083.3333\alpha}$ = \$.0602

Summary

Separated whey(6.5% TS) yield per lb cheese = α pound

Condensed whey yield per lb cheese = .1083α pound

Revenue per lb condensed whey = \$.0780

Cost per lb condensed whey = \$.0178

Profit per lb condensed whey = \$.0602

Exhibit 3.3-A. A model maximizing total profit contributions from a block Cheddar cheese vat

---

Objective Function

Maximize Z(=Total profit contributions from a block cheese vat output)

z = 1.3060*F(block) + .8219*F(crm-rem) + .7859*F(wheycrm) + .0602*F(condwhey) - .1197*X(milk) - .8235*X(crm-add) - .81*X(nfdm) - .239*X(condskim) - 378.3

Constraints

Capacity of a cheese vat (batch size)

X(milk) + X(crm-add) + X(nfdm) + X(condskim) - F(crm-rem) ≤ 30000

Lower level of casein to fat ratio(C/F ≥ .68)

29.13*F(crm-rem) + .064*X(milk) - 29.21*X(crm-add) + 27.32*X(nfdm) + 8.9484*X(condskim) ≥ 0

Upper level of casein to fat ratio(C/F ≤ .70)

30.03*F(crm-rem) - .01*X(milk) - 30.11*X(crm-add) + 27.3*X(nfdm) + 8.941*X(condskim) ≤ 0

Maximum amount of cream that can be removed from raw milk

F(crm-rem) ≤ .0822*X(milk)

Cheese yield per batch

100*F(block) = 10.2387*X(milk) + 74.7159*X(crm-add) + 48.1157*X(nfdm) + 15.3403*X(condskim) - 74.8487*F(crm-rem)

Whey cream yield per batch

100*F(wheycrm) = -7*F(crm-rem) + .5736*X(milk) + 7*X(crm-add) + .1556*X(nfdm) + .0576*X(condskim)

Condensed whey (60% TS) yield per batch

F(condwhey) = -1.9658*F(crm-rem) + 9.6590*X(milk) + 1.9802*X(crm-add) + 5.6022*X(nfdm) + 9.1624*X(condskim)

Nonnegativity constraints of the ingredients and finished and by-products for Cheddar cheese

X(milk), X(crm-add), X(nfdm), X(condskim) ≥ 0

F(block), F(crm-rem), F(wheycrm), F(condwhey) ≥0

Exhibit 3.3-B. Matrix form of the model of optimizing block Cheddar formulation

MAXIMIZE $Z = \underline{S}^t\underline{P} - 378.3$

$$= \begin{bmatrix} 1.3060 & .8219 & .7859 & .0602 & .1197 & .8235 & .81 & .239 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ -x_1 \\ -x_2 \\ -x_3 \\ -x_4 \end{bmatrix} - 378.3$$

SUBJECT TO

$$\begin{bmatrix} 0 & -29.13 & 0 & 0 & .064 & -29.21 & 27.32 & 8.9484 \\ 0 & 30.03 & 0 & 0 & .01 & 30.11 & -27.3 & -8.941 \\ 0 & 100 & 0 & 0 & 8.22 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ -x_1 \\ -x_2 \\ -x_3 \\ -x_4 \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 30000 \end{bmatrix}$$

$$\begin{bmatrix} 100 & 74.8487 & 0 & 0 & 10.2387 & 74.8487 & 48.1157 & 15.3403 \\ 0 & 7 & 100 & 0 & .5736 & 7 & .1556 & .0576 \\ 0 & 1.9658 & 0 & 100 & 9.659 & 1.9658 & 5.6022 & 9.1624 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ -x_1 \\ -x_2 \\ -x_3 \\ -x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{P}^t = \begin{bmatrix} f_1 & f_2 & f_3 & f_4 & -x_1 & -x_2 & -x_3 & -x_4 \end{bmatrix} \geq \underline{0}$$

<u>Objective solution</u>

$321.943

$$\text{at } \underline{P}^t = \begin{bmatrix} 3113.90 & 0 & 176.29 & 2892.67 & -29934.41 & -65.59 & 0 & 0 \end{bmatrix}$$

(Exhibit 3.3-B continued)

Where

$\underline{P}$ = a vector of product-mix and variable ingredient-mix,

f_i = the amount of output product i per vat,

f_1 = the amount of block Cheddar,

f_2 = the amount of cream removed,

f_3 = the amount of whey cream,

f_4 = the amount of condensed whey,

x_j = the amount of input resource j per vat,

x_1 = the amount of raw milk,

x_2 = the amount of cream added,

x_3 = the amount of nonfat dry milk,

x_4 = the amount of condensed skim milk;

$\underline{S}$ = a vector of profit contributions per pound output products, and cost per pound input resources,

s_1 = a profit contribution per pound block cheese,

s_2 = a profit contribution per pound cream removed,

s_3 = a profit contribution per pound whey cream,

s_4 = a profit contribution per pound condensed whey,

s_5 = cost per pound raw milk,

s_6 = cost per pound cream added,

s_7 = cost per pound nonfat dry milk,

s_8 = cost per pound condensed skim milk.

Exhibit 3.4-A. A model minimizing cheese manufacturing costs per pound cheese from a barrel Cheddar cheese vat

Objective Function

Minimize Z(=Cheese manufacturing cost per pound cheese)
Z = COSTS/F(barrel)

Constraints

Cost of manufacturing cheese and processing whey

COSTS = .002*F(barrel) + .0016*F(crm-rem) + .0016*F(wheycrm) + .0178*F(condwhey) + .1197*X(milk) + .8235*X(crm-add) + .8100*X(nfdm) + .2390*X(condskim) + 369.9

Capacity of a cheese vat (batch size)

X(milk) + X(crm-add) + X(nfdm) + X(condskim) - F(crm-rem) = 30000

Lower level of casein to fat ratio(C/F ≥ .68)

29.13*F(crm-rem) + .064*X(milk) - 29.21*X(crm-add) + 27.32*X(nfdm) + 8.9484*X(condskim) ≥ 0

Upper level of casein to fat ratio(C/F ≤ .70)

30.03*F(crm-rem) - .01*X(milk) - 30.11*X(crm-add) + 27.3*X(nfdm) + 8.941*X(condskim) ≤ 0

Maximum amount of cream that can be removed from raw milk

F(crm-rem) ≤ .0822*X(milk)

Cheese yield per batch

100*F(barrel) = - 76.056*F(crm-rem) + 10.4039*X(milk) + 75.921*X(crm-add) + 48.8917*X(nfdm) + 15.876*X(condskim)

Whey cream yield per batch

100*F(wheycrm) = -7*F(crm-rem) + .5736*X(milk) + 7*X(crm-add) + .1556*X(nfdm) + .0576*X(condskim)

Condensed whey (60% TS) yield per batch

F(condwhey) = - 1.8497*F(crm-rem) + 9.6411*X(milk) + 1.9179*X(crm-add) + 5.5182*X(nfdm) + 9.1044*X(condskim)

Nonnegativity constraints of the ingredients and finished and by-products for Cheddar cheese

X(milk), X(crm-add), X(nfdm), X(condskim) ≥ 0
F(block), F(crm-rem), F(wheycrm), F(condwhey) ≥ 0

Exhibit 3.4-B. Matrix form of the model of optimizing barrel Cheddar formulation

---

MINIMIZE $Z = (\underline{C}^t\underline{P} + 369.9)/f_1$

$$= \left(\begin{bmatrix} .002 & .0016 & .0016 & .0178 & .1187 & .8235 & .81 & .239 \end{bmatrix} \underline{P}^t + 369.9\right)/f_1$$

SUBJECT TO

$$\begin{bmatrix} 0 & -29.13 & 0 & 0 & .064 & -29.21 & 27.32 & 8.9484 \\ 0 & 30.03 & 0 & 0 & .01 & 30.11 & -27.3 & -8.941 \\ 0 & 100 & 0 & 0 & 8.22 & 0 & 0 & 0 \end{bmatrix} \underline{P}^t = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 100 & 76.0560 & 0 & 0 & 10.4039 & 75.9210 & 48.8917 & 15.8760 \\ 0 & 7 & 100 & 0 & .5736 & 7 & .1556 & .0576 \\ 0 & 1.8497 & 0 & 100 & 9.6411 & 1.8497 & 5.5182 & 9.1044 \\ 0 & -1 & 0 & 0 & -1 & -1 & -1 & -1 \end{bmatrix} \underline{P}^t = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 30000 \end{bmatrix}$$

$$\underline{P}^t = \begin{bmatrix} f_1 & f_2 & f_3 & f_4 & -x_1 & -x_2 & -x_3 & -x_4 \end{bmatrix} \geq \underline{0}$$

Objective solution

$1.285

at $\underline{P}^t = \begin{bmatrix} 3164.14 & 0 & 176.30 & 2887.23 & -29934.41 & -65.59 & 0 & 0 \end{bmatrix}$

(Exhibit 3.4-B continued)

Where:

$\underline{P}$ = a vector of product-mix and variable ingredient-mix,

f_i = the amount of output product i per vat,

f_1 = the amount of barrel Cheddar,

f_2 = the amount of cream removed,

f_3 = the amount of whey cream,

f_4 = the amount of condensed whey,

x_j = the amount of input resource j per vat,

x_1 = the amount of raw milk,

x_2 = the amount of cream added,

x_3 = the amount of nonfat dry milk,

x_4 = the amount of condensed skim milk;

$\underline{C}$ = a vector of cost per pound output products and input resources,

c_1 = cost per pound barrel cheese,

c_2 = cost per pound cream removed,

c_3 = cost per pound whey cream,

c_4 = cost per pound condensed whey,

c_5 = cost per pound raw milk,

c_6 = cost per pound cream added,

c_7 = cost per pound nonfat dry milk,

c_8 = cost per pound condensed skim milk.

Exhibit 3.5. An IP Model for Product Mix Optimization

MAXIMIZE $f(\underline{x},\underline{w},\underline{y})$ =

$6.5480X_{11} + 7.0480X_{12} + 6.9625X_{21} + 7.4625X_{22} + 4.3391X_{31} +$
$4.8391X_{32} + 4.4176X_{41} + 4.9176X_{42} + 4.4400X_{51} + 4.9400X_{52} +$
$3.6275X_{61} + 4.1275X_{62} + .011W_1 + .0602Y_1 + .7784Y_2$

SUBJECT TO

$13950 \le X_{11} \le 14229$
$7000 \le X_{12} \le 7140$
$5400 \le X_{21} \le 5508$
$2960 \le X_{22} \le 3019$
$5100 \le X_{31} \le 5254$
$1990 \le X_{32} \le 2030$
$5500 \le X_{41} \le 5610$
$3300 \le X_{42} \le 3366$
$4100 \le X_{51} \le 4182$
$2400 \le X_{52} \le 2448$
$4800 \le X_{61} \le 4896$
$2950 \le X_{62} \le 3009$
$X_{11} + X_{12} = 80B_1$
$X_{21} + X_{22} = 80B_2$
$X_{31} + X_{32} = 80B_3$
$X_{41} + X_{42} = 80B_4$
$X_{51} + X_{52} = 80B_5$
$X_{61} + X_{62} = 80B_6$
$B_1 + B_2 + B_3 + B_4 + B_5 + B_6 \le 750$
$10.5X_{11} + 10.5X_{12} + 11.55X_{21} + 11.55X_{22} + 10.5X_{31} + 10.5X_{32} +$
$10.325X_{41} + 10.325X_{42} + 10.5X_{51} + 10.5X_{52} + 10.5X_{61} + 10.5X_{62} \le W_2$
$W_1 - 3113.90V_1 = 0$
$W_2 - 3164.14V_2 = 0$
$V_1 + V_2 \le 720$
$Y_1 = .9292W_1 + .9125W_2$
$Y_2 = .0566W_1 + .0557W_2$
$Y_1 - 2.5X_{21} - 2.5X_{22} - 2.25X_{31} - 2.25X_{32} - 2.0X_{41} - 2.0X_{42} -$
$2.25X_{51} - 2.25X_{52} - 2.25X_{61} - 2.25X_{62} = WS$
X_{ij} integer, $i \in I$ and $j \in J$
B_i integer, $i \in I$
V_k integer, $k \in K$.

Table 3.11. Coding of products and resources for the cheese manufacture

Item No	Notation	Unit of Measure	Description of Products and Ingredients
100	**Packaged products**		
01	FD-08	case	50, 8 oz cups of process cheese food
02	FD-16	case	25, 16 oz cups of process cheese food
03	PN-08	case	50, 8 oz cups of plain cheese spread
04	PN-16	case	25, 16 oz cups of plain cheese spread
05	CO-08	case	50, 8 oz cups of chives/onion cheese spread
06	CO-16	case	25, 16 oz cups of chives/onion cheese spread
07	NR-08	case	50, 8 oz cups of nacho/red pepper cheese spread
08	NR-16	case	25, 16 oz cups of nacho/red pepper cheese spread
09	BH-08	case	50, 8 oz cups of bacon/hickory smoke cheese spread
10	BH-16	case	25, 16 oz cups of bacon/hickory smoke cheese spread
11	SH-08	case	50, 8 oz cups of salami/hickory smoke cheese spread
12	SH-16	case	25, 16 oz cups of salami/hickory smoke cheese spread
200	**Cheddar cheese**		
01	BLOCK	lb	block cheddar cheese
02	CHE-Y	lb	barrel cheddar cheese(young)
03	CHE-O	lb	aged cheddar cheese(old)
04	CHE-M	lb	aged cheddar cheese(medium)
300	**By-products**		
01	CRMRE	lb	cream removed from milk
02	WY-CR	lb	whey cream
03	CN-WY	lb	condensed whey

(Table 3.11. continued)

Item No	Notation	Unit of Measure	Description of Products and Ingredients
400	**Unpackaged products**		
01	CFD-B	unit	process cheese food batch
02	PLN-B	unit	plain cheese spread batch
03	C&O-B	unit	chives & onion cheese spread batch
04	N&R-B	unit	nacho & red pepper cheese spread batch
05	B&H-B	unit	bacon & hickory smoke cheese spread batch
06	S&H-B	unit	salmi & hickory smoke cheese spread batch
07	CH-FD	lb	process cheese food
08	PLN-S	lb	plain process cheese spread
09	C&O-S	lb	chives & onion cheese spread
10	N&R-S	lb	nacho & red pepper cheese spread
11	B&H-S	lb	bacon & hickory smoke cheese spread
12	S&H-S	lb	salmi & hickory smoke cheese spread
500	**Packaging materials**		
01	CASE	case	a case for 50, 8 oz. cups or 25, 16 oz. cups
02	CUP-A	cup	a 8 oz. cup with a cap
03	CUP-B	cup	a 16 oz. cup with a cap
04	PAKGE	unit	packaging material for 500 lb barrel Cheddar
05	PKGBK	unit	packaging material for 40 lb block Cheddar
600	**Cheese blends**		
01	F-BLN	lb	cheddar cheese blend for process cheese food
02	S-BLN	lb	cheddar cheese blend for cheese spread

(Table 3.11. continued)

Item No	Notation	Unit of Measure	Description of Products and Ingredients
700	**Milk ingredients** (for standardization)		
01	MILK	lb	milk
02	CREAM	lb	cream(45%)
03	NFDM	lb	non fat dry milk
04	CNDSK	lb	condensed skim milk
800	**Other ingredients**		
01	BUTER	lb	butter fat(80%)
02	CN-WY	lb	condensed whey
03	RENET	oz	rennet
04	START	lb	starter culture
05	COLOR	oz	color(annatto)
06	SALT	lb	salt
07	WPC	lb	whey protein concentrate
08	WATER	lb	water
09	EMULS	lb	emulsifiers
10	SALT	lb	salt
11	CHIVE	lb	dehydrated chive
12	ONION	lb	onion powder flavor
13	RDPEP	lb	red pepper
14	NACHO	lb	nacho flavor
15	BACON	lb	bacon bits
16	HIKOR	lb	hickory smoke flavor
17	SALAM	lb	salami

Table 3.12. A formulation of process cheese food[a]

Ingredients	Proportion of ingredients
Cheddar blend	60.0 %[b]
Butter fat(80%)	1.0
WPC	10.0
Water	16.5
Emulsifiers	2.0
Salt	0.5
Cheese food	100.0 %

[a] Federal Standards of Identity state that process cheese food must contain not more than 44% and not less than 23% milk fat.

[b] Cheddar cheese blends account for 60% young, 25% medium-aged, and 15% old-aged Cheddar.

Table 3.13. Formulations of process cheese spreads[a]

Ingredients	Plain	Chives & onion	Nacho R.Pepper	Bacon & H.Smoke	Salami& H.Smoke
Cheddar blend[b]	66.00%	60.00%	59.00%	60.00%	60.00%
Butter	8.40	6.00	5.80	6.30	6.20
Cond. whey	10.00	9.00	8.00	9.00	9.00
Water	18.50	18.60	18.50	18.50	18.00
Emulsifier	2.00	2.00	2.00	2.00	2.00
Salt	.50	.50	.50	.40	.40
Dehydrated Chives		.90			
Onion flavor		2.00			
Red peppers			4.60		
Nacho flavor			1.60		
Bacon bits				4.20	
Hikory smoke flavor				.50	.50
Salami					3.40
Total	100.00%	100.00%	100.00%	100.00%	100.00%

[a] Federal Standards of Identity state that process cheese spreads must contain not less than 44% and not more than 60% moisture, and not less than 20% milk fat.

[b] Cheddar cheese blends account for 70% young, 15% medium-aged, and 15% old-aged Cheddar.

	BLOCK	BAREL
BLOCK	1.0000	.0000
BAREL	.0000	1.0000
MILK	9.6132	9.4605
CREAM	.0211	.0207
RENET	.0289	.0284
START	.0674	.0664
COLOR	.0096	.0095
SALT	.0145	.0142
LABOR	.0057	.0057
ELECT	.1067	.0642
GAS	.0137	.0135
WY-CR	-.0566	-.0557
CN-WY	-.9289	-.9125

Figure 3.2. Per pound basis BOM matrix for Cheddar cheese

		CH-FD	PLN-S	C&O-S	N&R-S	B&H-S	S&H-S	F-BLN	S-BLN
	CH-FD	0	0	0	0	0	0	0	0
	PLN-S	0	0	0	0	0	0	0	0
	C&O-S	0	0	0	0	0	0	0	0
1	N&R-S	0	0	0	0	0	0	0	0
	B&H-S	0	0	0	0	0	0	0	0
	S&H-S	0	0	0	0	0	0	0	0
2	F-BLN	.700	0	0	0	0	0	0	0
	S-BLN	0	.66	.60	.59	.60	.60	0	0
	BUTER	.010	.084	.060	.058	.063	.062	0	0
	CN-WY	0	.10	.09	.08	.09	.09	0	0
	WPC	.100	0	0	0	0	0	0	0
	WATER	.165	.185	.186	.185	.185	.180	0	0
	EMULS	.020	.020	.020	.020	.020	.020	0	0
	SALT	.005	.005	.005	.005	.004	.004	0	0
3	CHIVE	0	0	.009	0	0	0	0	0
	ONION	0	0	.020	0	0	0	0	0
	RDPEP	0	0	0	.0046	0	0	0	0
	NACHO	0	0	0	.0160	0	0	0	0
	BACON	0	0	0	0	.0420	0	0	0
	HIKOR	0	0	0	0	.0050	.0050	0	0
	SALAM	0	0	0	0	0	.0420	0	0
	LABOR	.004	.004	.004	.004	.004	.004	0	0
	ELECT	.0662	.0662	.0662	.0662	.0662	.0662	0	0
	GAS	.012	.012	.012	.012	.012	.012	0	0
	CHE-O	0	0	0	0	0	0	.15	.15
4	CHE-M	0	0	0	0	0	0	.25	.15
	CHE-Y	0	0	0	0	0	0	.6	.70

Figure 3.3. Per pound basis recipe matrix **R** for the process cheese manufacture

The "‖" indicates the undescribed columns which have zero entries in the original 27 by 27 matrix.

	CH-FD	0	0	0	0	0	0	0	0	0
	PLN-S	0	0	0	0	0	0	0	0	0
	C&O-S	0	0	0	0	0	0	0	0	0
1	N&R-S	0	0	0	0	0	0	0	0	0
	B&H-S	0	0	0	0	0	0	0	0	0
	S&H-S	0	0	0	0	0	0	0	0	0
2	F-BLN	.700	0	0	0	0	0	0	0	0
	S-BLN	0	.66	.60	.59	.60	.60	0	0	0
	BUTER	.010	.084	.060	.058	.063	.062	0	0	0
	CN-WY	0	.10	.09	.08	.09	.09	0	0	0
	WPC	.100	0	0	0	0	0	0	0	0
	WATER	.165	.185	.186	.185	.185	.180	0	0	0
	EMULS	.020	.020	.020	.020	.020	.020	0	0	0
	SALT	.005	.005	.005	.005	.004	.004	0	0	0
3	CHIVE	0	0	.009	0	0	0	0	0	0
	ONION	0	0	.020	0	0	0	0	0	0
	RDPEP	0	0	0	.0046	0	0	0	0	0
	NACHO	0	0	0	.0160	0	0	0	0	0
	BACON	0	0	0	0	.0420	0	0	0	0
	HIKOR	0	0	0	0	.0050	.0050	0	0	0
	SALAM	0	0	0	0	0	.0420	0	0	0
	LABOR	.004	.004	.004	.004	.004	.004	0	0	0
	ELECT	.0662	.0662	.0662	.0662	.0662	.0662	0	0	0
	GAS	.012	.012	.012	.012	.012	.012	0	0	0
	CHE-O	0	0	0	0	0	0	.15	.15	0
4	CHE-M	0	0	0	0	0	0	.25	.15	0
	CHE-Y	0	0	0	0	0	0	.6	.70	0
	PKAGE	0	0	0	0	0	0	0	0	.0020
	MILK	0	0	0	0	0	0	0	0	9.4605
	CREAM	0	0	0	0	0	0	0	0	.0207
5	RENET	0	0	0	0	0	0	0	0	.0284
	START	0	0	0	0	0	0	0	0	.0664
	COLOR	0	0	0	0	0	0	0	0	.0095
	SALT	0	0	0	0	0	0	0	0	.0237
	LABOR	0	0	0	0	0	0	0	0	.0057
	ELECT	0	0	0	0	0	0	0	0	.0642
	GAS	0	0	0	0	0	0	0	0	.0135
6	WY-CR	0	0	0	0	0	0	0	0	-.0557
	CN-WY	0	0	0	0	0	0	0	0	-.9125

Figure 3.4. Per pound basis integrated recipe matrix $\mathbf{R}_I$ for the process cheese manufacture

The "‖" indicates the undescribed columns which have zero entries in the original 39 by 39 matrix.

		CH-FD	PLN-S	C&O-S	N&R-S	B&H-S	S&H-S	F-BLN	S-BLN
	CH-FD	1.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	PLN-S	.0000	1.0000	.0000	.0000	.0000	.0000	.0000	.0000
1	C&O-S	.0000	.0000	1.0000	.0000	.0000	.0000	.0000	.0000
	N&R-S	.0000	.0000	.0000	1.0000	.0000	.0000	.0000	.0000
	B&H-S	.0000	.0000	.0000	.0000	1.0000	.0000	.0000	.0000
	S&H-S	.0000	.0000	.0000	.0000	.0000	1.0000	.0000	.0000
2	F-BLN	.7000	.0000	.0000	.0000	.0000	.0000	1.0000	.0000
	S-BLN	.0000	.6600	.6000	.5900	.6000	.6000	.0000	1.0000
	BUTER	1.0000	.0840	.0600	.0580	.0630	.0620	.0000	.0000
	CN-WY	.0000	.1000	.0900	.0800	.0900	.0900	.0000	.0000
	WPC	.1000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	WATER	.1650	.1850	.1860	.1850	.1850	.1850	.0000	.0000
	EMULS	.0200	.0200	.0200	.0200	.0200	.0200	.0000	.0000
	SALT	.0050	.0050	.0050	.0050	.0040	.0040	.0000	.0000
3	CHIVE	.0000	.0000	.0090	.0000	.0000	.0000	.0000	.0000
	ONION	.0000	.0000	.0200	.0000	.0000	.0000	.0000	.0000
	RDPEP	.0000	.0000	.0000	.0460	.0000	.0000	.0000	.0000
	NACHO	.0000	.0000	.0000	.0160	.0000	.0000	.0000	.0000
	BACON	.0000	.0000	.0000	.0000	.0420	.0000	.0000	.0000
	HIKOR	.0000	.0000	.0000	.0000	.0050	.0050	.0000	.0000
	SALAM	.0000	.0000	.0000	.0000	.0000	.0340	.0000	.0000
	LABOR	.0040	.0040	.0040	.0040	.0040	.0040	.0000	.0000
	ELECT	.0662	.0662	.0662	.0662	.0662	.0662	.0000	.0000
	GAS	.0120	.0120	.0120	.0120	.0120	.0120	.0000	.0000
	CHE-O	.1050	.0990	.0900	.0885	.0900	.0900	.1500	.1500
4	CHE-M	.1750	.0990	.0900	.0885	.0900	.0900	.2500	.1500
	CHE-Y	.4200	.4620	.4200	.4130	.4200	.4200	.6000	.7000

Figure 3.5. Per pound basis BOM matrix **B** for the process cheese manufacture

Table 3.14. Profit contribution margins of process cheese products

Notation of product[a]	Direct production costs per case[b]	Profit contribution margin per case[c]
FD-08	$ 28.4520	$ 6.5480
FD-16	27.9520	7.0480
PN-08	27.8870	7.1130
PN-16	27.3870	7.6130
CO-08	30.5270	4.4730
CO-16	30.0270	4.9730
NR-08	30.4620	4.5380
NR-16	29.5620	5.0380
BH-08	30.4245	4.5755
BH-16	29.9245	5.0755
SH-08	31.2370	3.7630
SH-16	30.7370	4.2630

[a] The accurate name of the product corresponding to the notation is listed in Table 3.11.

[b] The direct production costs include raw material costs (ingredients, packaging materials), and other direct production costs (labor and utility, production supplies and maintenance costs) and were determined based on per 100 lb basis cost shown in Figure 4.1. The packaging costs are $3.15 and $2.65 per case of 50, 8 oz cups and 25, 16 oz cups, respectively.

[c] The selling prices of the products are the same to follow the general industry practices, regardless of direct production costs. The selling price per case is assumed $35.00.

Table 3.15. Decision variables in the model optimizing the product mix for the cheese manufacture

Notation[a]	unit	Decision variable
FD-08	case	X_{11}
FD-16	case	X_{12}
PN-08	case	X_{21}
PN-16	case	X_{22}
CO-08	case	X_{31}
CO-16	case	X_{32}
NR-08	case	X_{41}
NR-16	case	X_{42}
BH-08	case	X_{51}
BH-16	case	X_{52}
SH-08	case	X_{61}
SH-16	case	X_{62}
BLOCK	pound	W_1
CHE-Y	pound	W_2
CN-WY	pound	Y_1
WY-CR	pound	Y_2
Block cheese vat	each	V_1
Barrel cheese vat	each	V_2
FD cooker	each	B_1
PN cooker	each	B_2
CO cooker	each	B_3
NR cooker	each	B_4
BH cooker	each	B_5
SH cooker	each	B_6
Condensed whey sold	pound	WS

[a] The accurate name of the product corresponding to the notation is listed in Table 3.11.

Table 3.16. Young barrel Cheddar and condensed whey requirements of process cheese products (per case)

Notation of product[a]	Per case requirements	
	Young barrel	Condensed whey
FD-08	10.50	0.0
FD-16	10.50	0.0
PN-08	11.55	2.50
PN-16	11.55	2.50
CO-08	10.50	2.225
CO-16	10.50	2.225
NR-08	10.325	2.0
NR-16	10.325	2.0
BH-08	10.50	2.25
BH-16	10.50	2.25
SH-08	10.50	2.25
SH-16	10.50	2.25

[a] The accurate name of the product corresponding to the notation is listed in table 15.

Table 3.17. Monthly demand for process cheese products

Product	October	November	December
FD-08	13950	13900	13500
FD-16	7000	6960	6754
PN-08	5400	5280	5100
PN-16	2960	2970	2800
CO-08	5100	5120	5000
CO-16	1990	2080	1900
NR-08	5500	5500	5600
NR-16	3300	3205	3500
BH-08	4100	4110	4000
BH-16	2400	2430	2250
SH-08	4800	5360	5200
SH-16	2950	3420	3260

Table 3.18. Comparison of objective solutions between LP and IP optimization in terms of October demand

	LP	IP
Objective function value($):	577946.60	577730.10
Variables	Values	
X_{11}	14193.0000	14140.0000
X_{12}	7140.0000	7140.0000
X_{21}	5508.0000	5461.0000
X_{22}	3019.0000	3019.0000
X_{31}	5100.0000	5100.0000
X_{32}	1990.0000	2020.0000
X_{41}	5500.0000	5500.0000
X_{42}	3300.0000	3300.0000
X_{51}	4100.0000	4112.0000
X_{52}	2400.0000	2448.0000
X_{61}	4800.0000	4800.0000
X_{62}	2950.0000	2960.0000
W_1	1614715.0000	1613000.0000
W_2	637413.3000	639156.3000
Y_1	2082033.0000	2082030.0000
Y_2	126896.8000	126896.8000
V_1	518.5508	518.0000
V_2	201.4492	202.0000
B_1	266.6625	266.0000
B_2	106.5875	106.0000
B_3	88.6250	89.0000
B_4	110.0000	110.0000
B_5	81.2500	82.0000
B_6	96.8750	97.0000
WS	1995101.0000	1994990.0000

Table 3.19. Comparison of objective solutions between LP and IP optimization in terms of November demand

	LP	IP
Objective function value($):	575713.60	575396.30
Variables	Values	
X_{11}	13900.0000	13840.0000
X_{12}	6960.0000	6960.0000
X_{21}	5280.0000	5270.0000
X_{22}	2970.0000	2970.0000
X_{31}	5065.0000	5120.0000
X_{32}	2080.0000	2080.0000
X_{41}	5500.0000	5400.0000
X_{42}	3205.0000	3160.0000
X_{51}	4110.0000	4050.0000
X_{52}	2430.0000	2430.0000
X_{61}	5200.0000	5300.0000
X_{62}	3300.0000	3420.0000
W_1	1614985.0000	1613000.0000
W_2	637139.1000	639156.3000
Y_1	2082034.0000	2082030.0000
Y_2	126896.8000	126896.8000
V_1	518.6375	518.0000
V_2	201.3625	202.0000
B_1	260.7500	260.0000
B_2	103.1250	103.0000
B_3	89.3125	90.0000
B_4	108.8125	107.0000
B_5	81.7500	81.0000
B_6	106.2500	109.0000
WS	1994083.0000	1993910.0000

Table 3.20. Comparison of objective solutions between LP and IP optimization in terms of December demand

	LP	IP
Objective function value($):	574699.80	573487.70
Variables	Values	
X_{11}	13770.0000	13751.0000
X_{12}	6889.0000	6889.0000
X_{21}	5202.0000	5144.0000
X_{22}	2856.0000	2856.0000
X_{31}	5100.0000	5022.0000
X_{32}	1938.0000	1938.0000
X_{41}	5712.0000	5710.0000
X_{42}	3570.0000	3570.0000
X_{51}	4080.0000	4025.0000
X_{52}	2295.0000	2295.0000
X_{61}	5262.0020	5234.0000
X_{62}	3326.0000	3326.0000
W_1	1615283.0000	1616114.0000
W_2	636836.6000	635992.1000
Y_1	2082034.0000	2082036.0000
Y_2	126896.8000	126896.8000
V_1	518.7331	519.0000
V_2	201.2669	201.0000
B_1	258.2375	258.0000
B_2	100.7250	100.0000
B_3	87.9750	87.0000
B_4	116.0250	116.0000
B_5	79.6875	79.0000
B_6	107.3500	107.0000
WS	1993823.0000	1994336.0000

CHAPTER 4

APPLICATION OF THE PRODUCTION PLANNING FRAMEWORK TO A HYPOTHETICAL DAIRY PROCESSOR, CHEESE MANUFACTURER - PART III MATRIX DATA STRUCTURES APPLICATION

Matrix theory provides a sound base for developing a food production planning framework. Chapter 3 showed the matrix provides a reliable structure for organizing the data, and gozinto procedure (GP) is an analytical means to manipulate the matrix for acquiring the information on product resource requirements. In this chapter MDS as an application of the matrix theory is used to offer a consistent, flexible tool to manipulate the matrix for obtaining desired planning information and supporting management decisions. By using a computer and MDS, the data in a large matrix can be quickly stored, retrieved, and manipulated to measure the impact of the manipulation and derive the useful information. This chapter investigates and illustrates the potential applications of MDS to gain functional information for production planning.

MDS Application to BOM Matrix

The MDS application to the BOM matrix **B** provides valuable information for production planning, inventory control, purchasing, and product price management.

Conversion of Product and Resource Units

The food product BOM often requires resource requirement units to several decimal places because an accurate measurement of a material usage is critical to the assessment of production and inventory costs in the high volume and low margin food industry. If using many decimal places is inconvenient, the unit of the basis can be obtained by a multiplication of **B** by a desired number. If the user wants to avoid using many decimal places, the pound basis may be converted into different bases such as 100 pound or percentage basis. Figures 4.1 and 4.2 show per 100 pound basis BOM matrix $\mathbf{B}_{100}$ and integrated BOM matrix $\mathbf{B}_I$, respectively.

The convenient unit form of a product would vary with various purposes of departments. While marketing applications would mainly use packaged finished product unit for demand forecasting or distribution, accounting applications use packaged finished product unit or per pound basis for costing purposes. The manufacturing department would prefer a large volume unit, or a single batch unit if batching process is involved. The BOM matrix for packaged process cheese products **Bp** is shown in Figure 4.3. In this figure, stage levels of the process cheese manufacturing process are:

1 : packaged process cheese products,

2 : packaging materials of the packaged products,

3 : unpackaged finished process cheese products,

4 : intermediate products of the unpackaged finished products,

5 : resources added in the processing cooker,

6 : resources of the cheese blends.

Matrix multiplication is exercised to generate the information on per batch resource requirement for the process cheese manufacture as shown in Figure 4.4. The batch formula matrix is particularly useful to manufacturing because the matrix presents the exact amount of input resources going into the batch. Figure 4.5 presents package unit and batch unit bases BOM matrices for the Cheddar cheese manufacture.

The unit of an input resource form may vary with the stages of purchasing, processing, storage or distribution. If conversion relations among several forms of units are accurately defined and managed, each department can use its own unit basis without causing any confusion among them. These conversion relations can be incorporated into the matrix and managed by MDS.

The varied forms of product or resource units can lead to data redundancies in an organization. Data redundancies are one of the most common sources of errors in computer applications, confusion between various units, and errors in production planning and control, which are likely to result in inconsistent documents and reports and frustrate timely and correct decision making. For avoiding miscommunication among the departments and reducing the potential confusion and errors in planning and its implementation, the departments must have a consensus on definitions and coding of resources and units. Then, the departments can develop a BOM structure based on the definitions as presented in Table 3.11, and determine responsibilities for updating data consistently. The flexibility, organizational ability and computational speed of MDS can help build a common BOM structure and a data base system. By building the BOM structure, we can reduce greatly data redundancy and planning inconsistency, which permits centralized control over planning and control..

Product Cost and Composition

In a multi-product, multi-staged food processing plant, it is not simple to obtain accurate information on product cost. Computing the unit product cost is furthermore complicated when the batch process required for manufacturing multiple finished products is involved. A matrix can organize a large amount of data for products and their input resources. For example, unit costs and compositions of input resources shown in Table 4.1 are organized into a matrix, and MDS generated the information on production cost and product composition. A matrix **C** organizes unit cost, fat content, and moisture content of the input resources for process cheese products in Figure 4.6. By establishing the relationship between the product and its input resources through GP and organizing unit costs of input resources into the matrix, a complex task of computing the unit product cost can be easily performed by using MDS. Direct production costs per 100 lb product, and fat and moisture contents of the products were obtained by a multiplication of $\mathbf{C}^t$ by **B** as described in Figure 4.7. While the data about moisture and fat contents of the products are useful to check against government regulations, cost data are practical to promptly measure the impact of anticipated or actual changes of resource costs and recipe changes on product costs. Another advantage of using MDS is that MDS helps accurately measure the unit product cost by incorporating the production cost of the intermediate product internally required to manufacture the finished product. The production cost data can be used to compare with actual production costs and to track down and adjust cost variations. Having accurate cost data for each product would help management determine the selling price and marketing strategy of the product with a comparison to the performance of the product in the market.

Food processors characterized by a high volume and low margin business need to be fully aware of changing costs of input resource, and must cope with the changes by flexibly modifying product formulation, product mix, or product price to sustain a desired level of profits. By incorporating the changes into the matrix, MDS can help management maintain correct information on the product cost and composition when resource unit costs or resource requirements change. The subject of the effect of changes in business information on manufacturing and other functions are described in the later section of this chapter.

Total Resource Requirements and Costs for A Production Target

Total resource requirements for a production target over a certain time period can be obtained by MDS. In Figure 4.8, a matrix **Y** containing total resource requirements for each month are attained by a multiplication of Bp (BOM matrix for packaged process cheese products) by **S** (the production target for the fourth quarter). The total resource requirements for the quarter can be obtained by summing the columns. Figure 4.9 shows the batch requirement for the time period, which is useful to schedule the batch process. By having the information of the total resource requirements, management can measure how much it will cost to purchase the resources for a specific period of time. The accurate and timely measurement of the costs helps management effectively perform procurement planning of resources and its implementation, and improve communication with vendors. It also helps financial planning including the correct anticipation of working capital needs, which offers significant potential for improving corporate cash flow and return on asset.

MDS Application to Matrix R

Recipes and Costs for Individual Manufacturing Stages

The direct relationship between a parent item and its direct resources described in the recipe matrix **R** can be singled out in a submatrix. Actually, personnel working for a particular manufacturing stage may not have to know the entire recipe of the product. For instance, the workers in charge of blending Cheddar cheese need to know only the blend recipe. Likewise, the workers operating process cheese cookers may only need to know cooking recipes as illustrated in Figure 4.4. Thus, the recipe for each manufacturing stage can be expressed by **R**'s submatrices. The **R**'s submatrix organizing the recipe for each manufacturing stage can be used to obtain the operating cost of the particular stage through a multiplication of the recipe submatrix by the cost submatrix organizing the costs of the resources required in the stage. This computation is also functional to evaluate the alternatives for a particular manufacturing stage in terms of costs as well as resource requirements.

MDS Application to Matrix T

Computing Net Resource Requirements

In general, there are some stocks available at the beginning of a certain time period. When the inventories are involved, it is important to identify the net resource requirement of the demand during a certain period of time to obtain accurate purchasing, production, and financial planning. Mize, White and Brooks (78) suggested a matrix approach for computing the net resource requirement. It is found, however, that the approach, multiplication of total resource requirement matrix by net demand matrix, does not always produce correct resource requirement information. The incorrect information is attributed to the double-counting of the intermediate product stock not only as the intermediate product itself, but also as resources of the intermediate product. In other words, the intermediate-product resources are increased by the amount equivalent to the amount required by the intermediate product, without any real increase in inventories. This pseudo-increase of the resource stocks may result in critical errors of inventory tracking and consequently production planning. It is particularly true when a short term planning like a week or month is performed and plans are updated.

Food processors generally keep a certain amount of intermediate products for obtaining certain desired quality attributes of finished products, or for a buffer between purchasing, production and distribution. For example, natural cheese is ripened and stored to acquire desired quality attributes for selling or for process cheese manufacture. The intermediate products are often stored instead of perishable raw materials for seasonal supplies of raw materials or seasonal consumption of the products, or for uncertain demand of particular finished products requiring the intermediate products. The planning procedure that provides correct information irrespective of the availability of intermediate product inventories is suggested below.

A procedure to obtain the net resource requirement

Step 1: Create a demand matrix **D** and an inventory matrix **V**, and go to step 2.

D includes a desired ending inventory of products and resources, while **V** includes outstanding purchasing orders as well as the stock available at the beginning of the planning period.

Step 2: Build a net demand matrix **N** by subtracting **V** from **D**: $\mathbf{N} = \mathbf{D} - \mathbf{V}$. Go to step 3.

Step 3: Obtain a conditional net resource requirement matrix **F** by a multiplication of **T** by **N**: $\mathbf{F} = \mathbf{TN}$.

If the entries of all the rows representing intermediate products are positive, go to step 5.1. Otherwise go to step 4.

Step 4: Create **R**'s submatrix $\mathbf{R}_s$ comprising the rows of intermediate products. Multiply $\mathbf{R}_s$ by **F**: $\mathbf{P} = \mathbf{R}_s\mathbf{F}$. If any entry of the resulting matrix **P** has a negative value, go to step 5.2. Otherwise, go to step 5.1.

Step 5.1: The net requirement matrix is: $\mathbf{Y} = \mathbf{F} = \mathbf{TN}$.

Step 5.2: Create a null matrix **O** with the size the same as **R** and replace **O**'s columns representing the intermediate product(s) with negative value(s) by the corresponding columns of **R**. The resulting matrix is called **Z**. Multiply **Z** by **F**, and then subtract the resulting matrix from **F**. The net requirement matrix **Y** is:

$$\mathbf{Y} = \mathbf{F} - \mathbf{ZF} = (\mathbf{I} - \mathbf{Z})\mathbf{F} = (\mathbf{I} - \mathbf{Z})\mathbf{TN} = (\mathbf{I} - \mathbf{Z})\mathbf{T}(\mathbf{D} - \mathbf{V})$$

In the step 5.1, the formula of step 5.2 can be used to express **Y** as follows:

$$\mathbf{Y} = \mathbf{TN} = (\mathbf{I} - \mathbf{Z})\mathbf{TN} = (\mathbf{I} - \mathbf{Z})\mathbf{T}(\mathbf{D} - \mathbf{V}), \text{ where } \mathbf{Z} = \mathbf{O}.$$

Therefore, the formula, $\mathbf{Y} = (\mathbf{I} - \mathbf{Z})\mathbf{T}(\mathbf{D} - \mathbf{V})$, may be used in any case to generate the total net resource requirement.

The matrix **Y** provides the information about how many units of finished products should be produced (the rows representing finished products), of intermediate products are produced (positive numbers in the rows representing intermediate products), of resources are purchased (positive numbers in the rows representing resources) and of resources are in stock (negative numbers in the rows representing resources). The procedure is illustrated with simple, manageable examples of the frozen dessert production.

Processing stages for frozen desserts include blending and base formulation, pasteurization and homogenization, mix storage, flavor addition, and freezing. The example involves the following products and ingredients:

a. finished products: strawberry frozen dessert (SD), banana-strawberry frozen dessert (BSD);

b. intermediate products: strawberry dessert base (SB), banana dessert base (BB);

c. direct resources for finished products excluding intermediate products: nuts (NUT), strawberry flavor (SF), banana flavor (BF);

d. direct resources for intermediate products: banana flavor (BF), strawberry flavor (SF), ice milk mix (IM).

The recipe matrix **R** and the total resource requirement matrix **T** are described below.

R =

	SD	BSD	SB	BB	NUT	BF	SF	IM
SD	0							
BSD	0	0						
SB	2	2	0			0		
BB	0	1	0	0				
NUT	1	1	0	0	0			
BF	0	1	0	3	0	0		
SF	1	0	2	0	0	0	0	
IM	0	0	1	1	0	0	0	0

T =

	SD	BSD	SB	BB	NUT	BF	SF	IM
SD	1							
BSD	0	1						
SB	2	2	1			0		
BB	0	1	0	1				
NUT	1	1	0	0	1			
BF	0	4	0	3	0	1		
SF	5	4	2	0	0	0	1	
IM	2	3	1	1	0	0	0	1

Example 1

1. Create the demand matrix **D** for a particular time period and the inventory matrix **V** at the beginning of the period.

D =

1000	SD
1800	BSD
0	SB
0	BB
0	NUT
0	BF
0	SF
0	IM

V =

700	SD
400	BSD
6000	SB
4000	BB
1800	NUT
1800	BF
1600	SF
4000	IM

2. By subtracting **V** from **D**, a net demand matrix **N** is obtained.

N = **D** - **V** =

300	SD
1400	BSD
-6000	SB
-4000	BB
-1800	NUT
-1800	BF
-1600	SF
-4000	IM

3. The conditional total net resource requirement matrix **F** is obtained by a multiplication of **T** by **N**: **F** = **TN**.

$$\mathbf{F} = \mathbf{TN} = \begin{bmatrix} 300 \\ 1400 \\ -2600 \\ -2600 \\ -100 \\ -8200 \\ -6500 \\ -9200 \end{bmatrix} \begin{matrix} SD \\ BSD \\ SB \\ BB \\ NUT \\ BF \\ SF \\ IM \end{matrix}$$

Since the entries of SB and BB have negative values, go to step 4.

4. Create **R**'s submatrix $\mathbf{R}_s$ comprising the rows of intermediate products and multiply $\mathbf{R}_s$ by **F**: $\mathbf{P} = \mathbf{R}_s\mathbf{F}$.

$$\mathbf{R}_s\mathbf{F} = \begin{bmatrix} 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 300 \\ 1400 \\ -2600 \\ -2600 \\ -100 \\ -8200 \\ -6500 \\ -9200 \end{bmatrix} = \begin{bmatrix} -2,600 \\ -2,600 \end{bmatrix} = \mathbf{P}$$

Since all entries of **P** have negative values, go to step 5.2.

5.2. Create a null matrix O with the size as same as **R**, and replace O's columns representing the intermediate product(s) with negative value(s) by the corresponding columns of **R**. The resulting matrix **Z** is:

$$\mathbf{Z} = \begin{bmatrix} 0 \\ 0 & 0 & & & & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Subtract **Z** from an identity matrix **I** and then multiply (**I** - **Z**) by **F**. The net requirement matrix **Y** is:

$\mathbf{Y} = (\mathbf{I} - \mathbf{Z})\mathbf{F}$

$$= \begin{bmatrix} 1 \\ 0 & 1 & & & & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -3 & 0 & 1 \\ 0 & 0 & -2 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 300 \\ 1400 \\ -2600 \\ -2600 \\ -100 \\ -8200 \\ -6500 \\ -9200 \end{bmatrix} = \begin{bmatrix} 300 \\ 1400 \\ -2600 \\ -2600 \\ -100 \\ -400 \\ -1300 \\ -4000 \end{bmatrix}$$

When **Y** is compared with **F**, significant stock differences of the resources for intermediate products were observed. Such an enormous difference is attributed to the double-counting of intermediate-product stocks as resources. In **F**, for example, the amount of IM available at the end of the time period is 9,200, compared with 4,000 of **Y**. The amount of -9,200 units was obtained as follows: -4,000 + 1(-2,600) + 1(-2,600) = -9,200. This shows that the amounts of the stock of SB and BB available at the end of the first quarter were also counted by the amount of their resource IM equivalent to the required amount of IM by SB and BB. This double-counting was also taken for the other two resources as follows:

BF: -8,200 - 3(-2,600) = -400

SF: -6,500 - 2(-2,600) = -1,300.

Another example is employed to see what would happen if some of intermediate products have stocks more than the requirement of the products for a time period.

<u>Example 2</u>

1. Create the demand matrix **D** and the inventory matrix **V**.

D =

1000	SD
1800	BSD
0	SB
0	BB
0	NUT
0	BF
0	SF
0	IM

V =

700	SD
400	BSD
2000	SB
1500	BB
1800	NUT
1800	BF
1600	SF
4000	IM

2. By subtracting **V** from **D**, a net demand matrix **N** is obtained.

$$\mathbf{N} = \mathbf{D} - \mathbf{V} = \begin{bmatrix} 300 \\ 1400 \\ -2000 \\ -1500 \\ -1800 \\ -1800 \\ -1600 \\ -4000 \end{bmatrix} \begin{matrix} SD \\ BSD \\ SB \\ BB \\ NUT \\ BF \\ SF \\ IM \end{matrix}$$

3. The conditional total net resource requirement matrix **F** is obtained by a multiplication of **T** by **N**: **F** = **TN**.

$$\mathbf{F} = \mathbf{TN} = \begin{bmatrix} 300 \\ 1400 \\ 1400 \\ -100 \\ -100 \\ -700 \\ 1500 \\ -2700 \end{bmatrix} \begin{matrix} SD \\ BSD \\ SB \\ BB \\ NUT \\ BF \\ SF \\ IM \end{matrix}$$

Since the entry of BB has a negative value, go to step 4.

4. Create **R**'s submatrix $\mathbf{R}_s$ comprising the rows of intermediate products and multiply $\mathbf{R}_s$ by **F**: $\mathbf{P} = \mathbf{R}_s\mathbf{F}$.

$$\mathbf{R}_s\mathbf{F} = \begin{bmatrix} 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 300 \\ 1400 \\ 1400 \\ -100 \\ -100 \\ -700 \\ 1500 \\ -2700 \end{bmatrix} = \begin{bmatrix} 1,400 \\ -100 \end{bmatrix} = \mathbf{P}$$

Since an entry of **P** has a negative value, go to step 5.2.

5.2. Create a null matrix O with the size as same as **R**, and replace O's columns representing the intermediate product(s) with negative value(s) by the corresponding columns of **R**. The resulting matrix **Z** is:

$$
\mathbf{Z} = \left[\begin{array}{cccccccc}
0 & & & & & & & \\
0 & 0 & & & & & & \\
0 & 0 & 0 & & & 0 & & \\
0 & 0 & 0 & 0 & & & & \\
0 & 0 & 0 & 0 & 0 & & & \\
0 & 0 & 0 & 3 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{array}\right]
$$

Subtract **Z** from an identity matrix **I** and then multiply the resulting matrix by **F**. The net requirement matrix **Y** is:

$$
\mathbf{Y} = (\mathbf{I} - \mathbf{Z})\mathbf{F}
$$

$$
= \left[\begin{array}{cccccccc}
1 & & & & & & & \\
0 & 1 & & & & & & \\
0 & 0 & 1 & & & 0 & & \\
0 & 0 & 0 & 1 & & & & \\
0 & 0 & 0 & 0 & 1 & & & \\
0 & 0 & 0 & -3 & 0 & 1 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 1
\end{array}\right]
\left[\begin{array}{r}
300 \\ 1400 \\ 1400 \\ -100 \\ -100 \\ -700 \\ 1500 \\ -2700
\end{array}\right]
= \left[\begin{array}{r}
300 \\ 1400 \\ 1400 \\ -100 \\ -100 \\ -400 \\ 1500 \\ -2600
\end{array}\right]
$$

The matrices **Y** and **F** have different entries at BF and IM rows. Smaller stocks of intermediate products contributed to the smaller differences than the first case. Overcounted values of BF and IM in **F** are corrected in the step 5.2 as follows:

BF: -700 - 3(-100) = -400

IM: -2,700 - 1(-100) = -2,600.

For keeping track of the inventories and computing net resource requirement, the procedure may be exercised quarterly, monthly, weekly or for a smaller time span. The double-counting problem may often occur when the inventories are tracked every day or every week. Computing the net resource requirement for more than one period simultaneously should be avoided since inventories are continuously moved from the end of a period to the beginning of the next period and the resulting ending inventory of a previous period should be taken into account in the next period.

Safety stocks and reorder point reflecting the lead time can be incorporated into matrices to track the inventories and make purchasing decisions such as whether or not to order and how much to order. Having the information on the net requirement and inventories can improve the efficiency and bargaining power of the purchasing department with vendors.

The GP and MDS can be programmed for a computer operation to produce valuable planning information timely and properly in response to the changes in market situation and production policies.

MDS Application to Managing Changes in Business Information

Changes in product requirements, prices of input resources, or product mix have important consequences not only for production, but also for other functions and profits of the company. The accounting department may have to revise product costs, whereas the marketing department may revise the product mix, margins and prices. While the purchasing department should revise product ordering and probably the vendor list, the manufacturing department should update BOM, and production and inventory decisions. Since the decisions and actions of one department responding to the changes greatly influence other departments and corporate profits, the cooperation of the departments is essential to corrective actions for the changes, and to avoiding potential confusion and disruptions among the departments. As mentioned in chapter 1, the integrated database system and BOM are vital to achieve the cooperative and integrated responses to the changes, and correct flows of information and materials. MDS provides a flexible means for managing the changes in information. The MDS applications to the management of the information change in product requirement and input resource cost are described below.

Changes in Resource Requirements of Unit Product

Modifications in a product recipe or formulation lead to the changes not only in the BOM of the product, but also in product costs. Justifications for the product recipe modifications include: development of improved recipe, new ingredient or technology, changes in customers' food consumption trends, new restriction of legal authorities on ingredients, products or processing methods, use of substitutes due to limited market supplies or increased resource costs, new or value-added product development, and product deletion.

Changes in per unit product requirement for resources can be classified into those for direct resources, indirect resources, and intermediate products. It is assumed the direct and indirect resources do not include intermediate products which have their own direct resource(s). Indirect resources of finished products are defined as direct resources of intermediate products.

Changes in the product requirement for the direct resource

When the recipe modification occurred to the resource which is directly required by a finished product(s), the BOM matrix **B** can be altered by simply replacing the original requirement with the revised one. For instance, if 100 lbs of process cheese food require 2.20 lbs of emulsifier instead of 2.00 lbs, **B** is altered by replacing 2.00 with 2.20 at $b_{EMULS,CH\text{-}FD}$ in Figure 4.1.

Changes in the product requirement for the indirect resource

When the recipe modification happened to the indirect resource of the finished product, the resource requirement of every finished product using the intermediate product should accordingly change. Suppose per unit (lb) young Cheddar cheese requirement for milk and cream changes from 9.4605 and .0207 to 9.4810 and .0, respectively. Since every process cheese product uses young Cheddar, per unit product requirement for milk and cream must consequently change. Per 100 lbs product requirement for milk and cream is obtained by the following steps:

1. Create the vector $\underline{r}$ of the revised young Cheddar requirements for milk and cream.

	original	revised		net change	
MILK	9.4605	9.4810	= $\underline{r}$	.0205	= $\underline{d}$
CREAM	0.0207	0.0		-.0207	

2. Build the submatrix **Y** of the product requirement for young Cheddar.

CH-FD	PLN-S	C&O-S	N&R-S	B&H-S	S&H-S	F-BLN	S-BLN
42.00	46.20	42.00	41.30	42.00	42.00	60.00	70.00

= **Y**

3. Multiply $\underline{r}$ by **Y** to obtain the matrix of per 100 lbs product requirement for milk and cream.

$\underline{r}$ x **Y** =

MILK	9.481
CREAM	0.0

CH-FD	PLN-S	C&O-S	N&R-S	B&H-S	S&H-S	F-BLN	S-BLN
42.00	46.20	42.00	41.30	42.00	42.00	60.00	70.00

=	CH-FD	PLN-S	C&O-S	N&R-S	B&H-S	S&H-S	F-BLN	S-BLN
MILK	398.2	438.02	398.2	391.57	398.2	398.2	568.86	663.67
CREAM	.0	.0	.0	.0	.0	.0	.0	.0

= **S**

4. Revise **B** by replacing **B**'s corresponding part with this resulting submatrix **S**.

The other way to revise **B** is to compute the net changes in per unit product requirement for milk and cream and add it to the original values. The net changes can be determined by a multiplication of a vector $\underline{d}$ containing the net difference between the revised requirement and the original requirement, and **Y**:

$\underline{d} \times \mathbf{Y}$ =

MILK	.0205
CREAM	-.0207

CH-FD	PLN-S	C&O-S	N&R-S	B&H-S	S&H-S	F-BLN	S-BLN
42.00	46.20	42.00	41.30	42.00	42.00	60.0	70.0

	CH-FD	PLN-S	C&O-S	N&R-S	B&H-S	S&H-S	F-BLN	S-BLN	
=MILK	.861	.947	.861	.847	.861	.861	1.230	1.435	$= \mathbf{S}_n$
CREAM	-.869	-.956	-.869	-.855	-.869	-.869	-1.242	-1.449	

Then, **B** is revised by adding the submatrix $\mathbf{S}_n$ to **B**'s corresponding part.

<u>Changes in the product requirement for the intermediate product</u>

When a new product recipe requires the changes in per unit product requirement for an intermediate product, the BOM modification should be exercised not only to per unit product requirement for the intermediate product, but also to per unit product requirement for the direct resources of the intermediate product, and indirect resources of the intermediate product if the intermediate product has any lower level intermediate product as its direct resource. For instance, suppose the process cheese spread product requirement is altered for Cheddar cheese blends in the integrated BOM as shown in Figure 4.2. Then, the product requirement for the different aged Cheddar cheeses and for the resources of young Cheddar should consequently change. Young Cheddar is not only a direct resource of the cheese blend, but also has its direct resources as an intermediate product in this example. The changes can be attained as follows:

1. Build the submatrix $\mathbf{S}_r$ organizing the revised young Cheddar requirements for milk and cream.

PLN-S	C&O-S	N&R-S	B&H-S	S&H-S
66.00	60.00	59.00	60.00	60.00

: submatrix $\mathbf{S}_0$ organizing the original requirement for cheese blend

62.00	60.10	58.20	59.00	59.50

: submatrix $\mathbf{S}_r$ organizing the revised requirement

-4.00	0.10	-0.80	-1.00	-0.50

: submatrix $\mathbf{S}_d$ organizing the net difference

2. Create submatrix **R** organizing per 100lb cheese spread blend requirement for young, medium, old aged cheeses and young Cheddar resources:

R =

15.00	CHE-O
15.00	CHE-M
70.00	CHE-Y
662.24	MILK
1.45	CREAM
1.99	RENET
4.65	START
0.67	COLOR
1.66	SALT
0.14	LABOR
4.50	ELECT
0.95	GAS
-3.90	WY-CR
-63.88	CN-WY

3. By a multiplication of **R** by $\mathbf{S}_r$, matrix **Q** organizing the revised per 100 lb cheese spread blend requirement for its resources is attained:

(**R** x $\mathbf{S}_r$)/100 =

15.00	62.00	60.10	58.20	59.00	59.50
15.00					
70.00					
662.24					
1.45					
1.99					
4.65					
.67					
1.66					
0.14					
4.50					
0.95					
-3.90					
-63.88					

$\frac{1}{100}$

product requirement for cheese spread resources

	PLN-S	C&O-S	N&R-S	B&H-S	S&H-S	
	9.30	9.02	8.73	8.85	8.93	CHE-O
	9.30	9.02	8.73	8.85	8.93	CHE-M
	43.40	42.07	40.74	41.30	41.65	CHE-Y
	410.59	403.97	385.42	390.72	394.03	MILK
	.90	.87	.84	.86	.86	CREAM
	1.23	1.20	1.16	1.17	1.18	RENET
	2.88	2.79	2.71	2.74	2.77	START
=	.42	.40	.39	.40	.40	COLOR
	1.03	1.00	.97	.98	.99	SALT
	.09	.08	.08	.08	.08	LABOR
	2.79	2.70	2.62	2.66	2.68	ELECT
	.59	.57	.55	.56	.57	GAS
	-2.42	-2.34	-2.27	-2.30	-2.32	WY-CR
	-39.61	-38.39	-37.18	-37.69	-38.01	CN-WY

The resulting matrix as a submatrix of **B** shows the revised product requirement for the direct and indirect resources of the cheese spread blend, and replaces a corresponding submatrix of **B**. Likewise, the net changes in the product requirement for the cheese spread blend resources can be determined by a product of **R** and $\mathbf{S}_d$.

Changes in the product requirement for multiple intermediate products in several levels

When it is necessary to simultaneously modify many levels of the product BOM, it may be efficient to modify the Recipe matrix **R** and then use GP to make a new BOM matrix. It is, however, important to note that net change matrix Δ**T** can not be obtained by applying the net change recipe matrix

ΔR to the Gozinto procedure. In other words, $(I - \Delta R)^{-1} \neq \Delta T$. Let R' and T' be a revised recipe matrix and a revised total requirement matrix, respectively, such that $R' = R + \Delta R$ and $T' = T + \Delta T$. Then, $T' = T + \Delta T = (I - R)^{-1} + \Delta T = (I - R')^{-1} + \Delta T = (I - R - \Delta R)^{-1}$. If $(I - \Delta R)^{-1} = \Delta T$, then $T' = (I - R - \Delta R)^{-1} = (I - R)^{-1} + (I - \Delta R)^{-1}$. However, this equation is not true. For example, suppose per lb cheese food requirement for cheese blend dropped from 0.70 Lb to 0.65 Lb, and the blend composition of cheddar cheeses changed from 15:25:60 to 20:25:55, respectively. The net change recipe matrix, ΔR, is described below:

ΔR =

- .05		F-BLN
	+ .05	CHE-O
	.0	CHE-M
	- .05	CHE-Y

The following computation example shows that $(I - \Delta R)^{-1}$ does not generate ΔT. In the revised BOM matrix **B**, the correct process cheese food requirement for CHE-O is 0.13 Lb as follows:

(.70-.05)(.15+.05)=.70(.15)-.05(.15)+.70(.05)-.05(.05)=.13

: Original requirement(.1050 Lb)

To produce 0.13 Lb in the revised **B**, (**I** - ▲**R**)$^{-1}$ should produce: -.05(.15) + .70(.05) - .05(.05) = .025. But (**I** - ▲**R**)$^{-1}$ generates: -.05(.05) = -.0025. Thus, it is easily known that (**I** - ▲**R**)$^{-1}$ is not equal to ▲**T**. Similarly, when ▲**R** is made as below, (**I** - ▲**R**)$^{-1}$ produce: -.05(.20) = .01, which is not the correct answer.

▲R =

- .05			F-BLN
		.20	CHE-O
		.25	CHE-M
		.55	CHE-Y

To generate the revised BOM matrix when product recipe modifications occur to many items including intermediate product and its resource composition, modified recipe matrix or the submatrix manipulation may be used.

Changes in Unit Product Cost by Recipe Modification

The recipe alteration changes the unit product cost. Food processors' high volume and low profit margin highlight the prompt response to the anticipated or actual changes in product recipes or input resource prices, and their impact on the product cost or margin in order to sustain their competitive position. MDS serves timely and convenient evaluation for their impact on product costs. The net changes in unit product costs according to the recipe modification can be obtained by taking a product of a vector for per unit cost of the resource whose unit requirement changed, and a matrix for the net changes in the product requirement for the resource. In the previous example of the net changes in per lb young Cheddar cheese requirement for milk and cream, the net changes in unit product costs can be obtained by a multiplication of a matrix for per lb price of milk and cream, and $\mathbf{S}_n$:

MILK	CREAM
.1197	.8235

x

CH-FD	PLN-S	C&O-S	N&R-S	B&H-S	S&H-S	F-BLN	S-BLN
.861	.947	.861	.847	.861	.861	1.230	1.435
-.869	-.956	-.869	-.855	-.869	-.869	-1.242	-1.449

=

CH-FD	PLN-S	C&O-S	N&R-S	B&H-S	S&H-S	F-BLN	S-BLN
-.613	-.602	-.613	-.603	-.613	-.613	-.876	-1.022

=Q

The resulting matrix **Q** indicates net cost savings per 100 lb products by the revised recipe. The revised unit product cost can be obtained by adding **Q** to the direct production cost matrix, or by taking a product of resource cost matrix and the revised product requirement matrix.

<u>Changes in Unit Product Cost by Modified Unit Resource Cost</u>

Changes in the unit prices of input resources may occur when planning production and/or during a production period. These changes that result in altering unit product costs and profit margins are primarily induced by the changes in market supply and demand conditions, management's costing policy, or vendor's minimum order quantity or pricing policy. For example, suppose the unit prices of red pepper and salami increased by $0.10 and $0.25, and the unit price of bacon decreased by $0.15. The impact of the variations on material costs of 100 lb process cheese products can be derived through the following matrix manipulation procedure:

1. Create revised cost matrix $\mathbf{C}_1$ by replacing the original unit costs of the resources with the altered unit costs in the unit cost matrix $\mathbf{C}_0$. The matrices $\mathbf{C}_0$ and $\mathbf{C}_1$ are described in Figure 4.10.
2. Multiply $\mathbf{C}_1$ by $\mathbf{B}$.

Revised material costs per 100 lb products are:

C_1^tB =

CH-FD	PLN-S	C&O-S	N&R-S	B&H-S	S&H-S	F-BLN	S-BLN	CHE-Y
[100.75	98.49	109.05	109.25	108.01	112.74	133.52	129.94	119.2]^t

The changes in input resource prices result in the changes in production costs of 3 product families (N&R-S, B&H-S, S&H-S). The net changes in unit product costs can be obtained by subtracting the original costs from the revised costs.

	CH-FD	PLN-S	C&O-S	N&R-S	B&H-S	S&H-S	F-BLN	S-BLN	CHE-Y
	[100.75	98.49	109.05	108.79	108.64	111.89	133.52	129.94	119.20]^t
-	[100.75	98.49	109.05	109.25	108.01	112.74	133.52	129.94	119.20]^t

	--------			N&R-S	B&H-S	S&H-S	-------		
=	[0	0	0	.46	-.63	.85	0	0	0]^t

Net changes in unit product costs can be also directly acquired. As described in Figure 4.10, create a matrix named C_v, of the same dimension as **C** with entries of the variances in the unit resource costs. Then, multiplying C_v by **B** produces the change in the costs of production:

	--------			N&R-S	B&H-S	S&H-S	-------		
C_v^tB =	[0	0	0	.46	-.63	.85	0	0	0]^t

Similarly, changes in the unit production costs of packaged products can be obtained. Changes in unit profit margins of the products can be determined by subtracting the unit product cost matrix from the unit selling price matrix, which would affect the marketing's product portfolio management. The matrix manipulation can be used to evaluate the bids of several vendors or to measure potential price changes in advance. Once the changes in business information and their impacts on the functional operations of the company are measured, the data must be accurately and promptly entered in the information system to generate accurate product BOM, costs and material requirement, and develop product price and mix to help the management respond to market conditions and formulate the business plans.

MDS Application to Quality Control

Food processors have long recognized the need for high quality products to meet consumer demand. The perishable nature of raw materials and products has made quality control extremely important during processing, storage, transportation, and even consumer handling. In addition, variability commonly occurs in manufacturing processes in the food industry because raw materials possess wide variability in their quality attributes. People, equipment, processing conditions, and test methods also contribute to the variability. Raw materials, production supplies, intermediates, and processes have several critical attributes or quality factors that affect physical, biological, chemical and functional properties of finished products. To reduce the level of variation in output product quality and improve productivity, quality factors should be monitored and manufacturing process continuously adjusted.

To achieve the goals of quality control, large amounts of data are collected, and used differently according to various purposes, including process control, analysis, inspection, or regulation. For example, a food processor may collect the data regarding % moisture, % fat, acidity, weight, number of microorganisms, and so on. Lot tracking of raw materials from purchasing to processing and products from processing to distribution may also be needed for quality control and government regulations. Data collection and evaluation will serve as the basis for proper decisions and actions. MDS is functional to monitoring the quality control of material flows by storing and evaluating the data. The most desirable feature of MDS is that MDS makes it easy to compile data in such a form that the data may be used in a timely manner and analyzed by computer.

Monitoring Quality Factors

The following simple example shows a function of MDS in quality control. Suppose each sample was taken from 3 batches of process cheese food according to a sampling method. In this example, the quality factors are set as pH, cooking temperature (°F), % moisture, and % fat. Acceptable ranges of numerical values for the factors are described in vectors $\underline{l}$ and $\underline{u}$, which represent lower limit and upper limit, respectively.

$$\underline{l} = \begin{bmatrix} 5.2 \\ 175.0 \\ 41.0 \\ 23.5 \end{bmatrix} \qquad \underline{u} = \begin{bmatrix} 5.6 \\ 185.0 \\ 43.5 \\ 25.0 \end{bmatrix} \quad \begin{matrix} \text{pH} \\ \text{Temperature(°F)} \\ \text{Moisture(\%)} \\ \text{Fat(\%)} \end{matrix}$$

Data from the samples are contained in vectors $\underline{x}_1$, $\underline{x}_2$ and $\underline{x}_3$:

$$\underline{x}_1 = \begin{bmatrix} 5.4 \\ 179.0 \\ 42.3 \\ 24.1 \end{bmatrix} \qquad \underline{x}_2 = \begin{bmatrix} 5.1 \\ 178.0 \\ 42.6 \\ 24.3 \end{bmatrix} \qquad \underline{x}_3 = \begin{bmatrix} 5.3 \\ 180.1 \\ 43.3 \\ 24.7 \end{bmatrix} \quad \begin{matrix} \text{pH} \\ \text{Temp(°F)} \\ \text{Moisture} \\ \text{Fat} \end{matrix}$$

By checking the values of a specific sample against the range, we can determine whether or not the sample is acceptable. In this case, the inspection is done by computing the difference between sample values and limit values: sample values minus lower limit values, and upper limit values minus sample values. Then, the status of the sample is determined. If all differences are nonnegative, the sample is accepted. The following example shows that sample 1 is accepted because every value of difference is nonnegative.

$$\underline{x}_1 - \underline{l} = \begin{bmatrix} 0.2 \\ 4.0 \\ 1.3 \\ 0.6 \end{bmatrix} \qquad \underline{u} - \underline{x}_1 = \begin{bmatrix} 0.2 \\ 6.0 \\ 1.2 \\ 0.9 \end{bmatrix}$$

Sample 2, on the other hand, is not accepted because the pH value is lower than the lower limit. This inspection can be easily computerized with automatic data entry and computation. The food processor would have standards of product and manufacturing process with the most desirable value (target value) for each quality factor. By computing the difference between sample value and target value and checking it against tolerable levels, the status of the sample can be also determined. Monitoring the quality factors can be extended to the individual stages of the manufacturing process, while reducing tolerable levels will eventually help improve quality.

Computing the Mean and Variance Using MDS

Statistics provide a way to analyze numerical data. When the distribution of the samples is investigated, individual data are not primarily important. In general, mean and variance are used to investigate the distribution of the samples. A generalized procedure for computing the mean and variance using MDS are described as below.

A sample vector $\underline{x}_j$ from a batch (or lot) j is:

$$\underline{x}_j = \begin{bmatrix} x_{1j} \\ x_{2j} \\ | \\ | \\ x_{mj} \end{bmatrix}$$

,where m = number of quality factors that are controlled.

The vectors $\underline{x}$'s can be organized into a matrix **X** to compute the mean and sample variance for each factor.

A matrix **X** containing n samples and their numerical values for the factors is:

$$\mathbf{X} = \begin{bmatrix} x_{11} & - - - & x_{1n} \\ x_{21} & - - - & x_{2n} \\ | & & | \\ | & & | \\ x_{m1} & - - - & x_{mn} \end{bmatrix} = (x_{ij})_{mxn}$$

Then, the matrix **X** can be divided into m vectors corresponding to the number of quality factors.

$$\underline{v}_1 = [\ x_{11} \quad x_{12} \quad - - - \quad x_{1n}\ ]^t$$

\- - - - - -

$$\underline{v}_m = [\ x_{m1} \quad x_{m2} \quad - - - \quad x_{mn}\ ]^t .$$

An elementary vector with n entries is:

$$\underline{i} = [\ 1 \quad 1 \quad 1 \quad - - - \quad 1\ ]^t .$$

A mean y_i of the values for a factor i is:

$$y_i = \frac{\underline{v}_i^{\,t} \cdot \underline{i}^t}{n} \quad \text{or} \quad \frac{\underline{v}_i^{\,t}\ \underline{i}}{n} .$$

A vector $\underline{y}$ containing the means for the factors is:

$$\underline{y} = [\ y_1 \quad y_2 \quad - - - - \quad y_m\ ]^t.$$

A sample variance s_i^2 of the values for a factor i is determined by :

$$s_i^2 = \frac{\sum_j^n (\underline{x}_{ij} - y_i)^2}{n - 1} = \frac{\sum_j^n \underline{x}_{ij}^2 - \frac{1}{n}(\sum_j^n x_{ij})^2}{n - 1}$$

$$= \frac{\sum_j^n \underline{x}_{ij}^2 - y_i \sum_j^n x_{ij}}{n - 1} , \qquad i = 1,\ 2,\ - - -,\ m$$

Also,

$$\sum_j^n \underline{x}_{ij}^2 = \underline{v}_i^{\,t} \cdot \underline{v}_i^{\,t} \quad \text{or} \quad \underline{v}_i^{\,t}\ \underline{v}_i ,$$

$$\sum_j^n \underline{x}_{ij} = \underline{v}_i^{\,t} \cdot \underline{i}^t \quad \text{or} \quad \underline{v}_i^{\,t}\ \underline{i}.$$

Therefore,

$$s_i^2 = \frac{\underline{v}_i^t \cdot \underline{v}_i^t - y_i(\underline{v}_i^t \cdot \underline{i}^t)}{n - 1} \quad \text{or} \quad \frac{\underline{v}_i^t\ \underline{v}_i - y_i(\underline{v}_i^t\ \underline{i})}{n - 1} \quad .$$

Then, a vector $\underline{s}$ containing the standard deviations for the factors is organized:

$$\underline{s} = [\ s_1 \quad s_2 \quad - - - - \quad s_m\ ]^t.$$

Similarly, mean values and standard deviations of the differences between the sample values and target values can be computed. A vector organizing the target values for quality factors is:

$$\underline{g} = [\ g_1 \quad g_2 \quad - - - - \quad g_m\ ]^t.$$

The difference between sample value and target value are organized in a matrix **D**:

$$\mathbf{D} = \mathbf{X} - \underline{g}^t \underline{i}.$$

Then, mean and standard deviation are computed and organized in vectors.

A histogram is an efficient way to arrange the data when there are many samples so it is difficult to determine the distribution of measurements by looking at the data (55). By constructing a histogram based on the data for each quality factor, it will be easy to identify the shape, central value, and the manner of dispersion of the measurement associated with the acceptable (tolerable) range. When raw materials, processing methods, workers, or equipment change during a certain period of time, it is imperative to know the effect of the change on material flows or quality characteristics of the product. When combined with a graphic feature like control charts, MDS will provide a convenient way to evaluate the changes and take suitable actions.

The applications of MDS to quality control can be used when attempting to improve yield, to reduce defects and quality variance, to investigate the relationship between cause and effect, and to study abnormal data. Many food processors now use advanced instruments and quality control systems. Computers and programmable controllers monitor electronic signals from processing on a real-time basis. A computerized system equipped with MDS logic will enable a food processor to observe the problem, identify the possible source of the problem, and take a necessary action before the problem becomes more serious. This procedure is valuable not only in continuous process, but also in batch processes because a defect of a batch will waste all outputs from the batch.

		CH-FD	PLN-S	C&O-S	N&R-S	B&H-S	S&H-S	F-BLN	S-BLN
	CH-FD	100.00	.00	.00	.00	.00	.00	.00	.00
	PLN-S	.00	100.00	.00	.00	.00	.00	.00	.00
1	C&O-S	.00	.00	100.00	.00	.00	.00	.00	.00
	N&R-S	.00	.00	.00	100.00	.00	.00	.00	.00
	B&H-S	.00	.00	.00	.00	100.00	.00	.00	.00
	S&H-S	.00	.00	.00	.00	.00	100.00	.00	.00
2	F-BLN	70.00	.00	.00	.00	.00	.00	100.00	.00
	S-BLN	.00	66.00	60.00	59.00	60.00	60.00	.00	100.00
	BUTER	1.00	8.40	6.00	5.80	6.30	6.20	.00	.00
	CN-WY	.00	10.00	9.00	8.00	9.00	9.00	.00	.00
	WPC	10.00	.00	.00	.00	.00	.00	.00	.00
	WATER	16.50	18.50	18.60	18.50	18.50	18.50	.00	.00
	EMULS	2.00	2.00	2.00	2.00	2.00	2.00	.00	.00
	SALT	.50	.50	.50	.50	.40	.40	.00	.00
3	CHIVE	.00	.00	.90	.00	.00	.00	.00	.00
	ONION	.00	.00	2.00	.00	.00	.00	.00	.00
	RDPEP	.00	.00	.00	4.60	.00	.00	.00	.00
	NACHO	.00	.00	.00	1.60	.00	.00	.00	.00
	BACON	.00	.00	.00	.00	4.20	.00	.00	.00
	HIKOR	.00	.00	.00	.00	.50	.50	.00	.00
	SALAM	.00	.00	.00	.00	.00	3.40	.00	.00
	LABOR	.40	.40	.40	.40	.40	.40	.00	.00
	ELECT	6.62	6.62	6.62	6.62	6.62	6.62	.00	.00
	GAS	1.20	1.20	1.20	1.20	1.20	1.20	.00	.00
	CHE-O	10.50	9.90	9.00	8.85	9.00	9.00	15.00	15.00
4	CHE-M	17.50	9.90	9.00	8.85	9.00	9.00	25.00	15.00
	CHE-Y	42.00	46.20	42.00	41.30	42.00	42.00	60.00	70.00

Figure 4.1. Per 100 pound basis BOM matrix B_{100} for the process cheese manufacture[a]

[a] The 100 lb-basis BOM matrix for process cheese products can be also interpreted as 100 lb-based BOM matrix. The sum of the levels 2 and 3 except labor and utilities is 100 %.

		CH-FD	PLN-S	C&O-S	N&R-S	B&H-S	S&H-S	F-BLN	S-BLN	CHE-Y
	CH-FD	100.00	.00	.00	.00	.00	.00	.00	.00	.00
	PLN-S	.00	100.00	.00	.00	.00	.00	.00	.00	.00
1	C&O-S	.00	.00	100.00	.00	.00	.00	.00	.00	.00
	N&R-S	.00	.00	.00	100.00	.00	.00	.00	.00	.00
	B&H-S	.00	.00	.00	.00	100.00	.00	.00	.00	.00
	S&H-S	.00	.00	.00	.00	.00	100.00	.00	.00	.00
2	F-BLN	70.00	.00	.00	.00	.00	.00	100.00	.00	.00
	S-BLN	.00	66.00	60.00	59.00	60.00	60.00	.00	100.00	.00
	BUTER	1.00	8.40	6.00	5.80	6.30	6.20	.00	.00	.00
	CN-WY	.00	10.00	9.00	8.00	9.00	9.00	.00	.00	.00
	WPC	10.00	.00	.00	.00	.00	.00	.00	.00	.00
	WATER	16.50	18.50	18.60	18.50	18.50	18.50	.00	.00	.00
	EMULS	2.00	2.00	2.00	2.00	2.00	2.00	.00	.00	.00
	SALT	.50	.50	.50	.50	.40	.40	.00	.00	.00
3	CHIVE	.00	.00	.90	.00	.00	.00	.00	.00	.00
	ONION	.00	.00	2.00	.00	.00	.00	.00	.00	.00
	RDPEP	.00	.00	.00	4.60	.00	.00	.00	.00	.00
	NACHO	.00	.00	.00	1.60	.00	.00	.00	.00	.00
	BACON	.00	.00	.00	.00	4.20	.00	.00	.00	.00
	HIKOR	.00	.00	.00	.00	.50	.50	.00	.00	.00
	SALAM	.00	.00	.00	.00	.00	3.40	.00	.00	.00
	LABOR	.40	.40	.40	.40	.40	.40	.00	.00	.00
	ELECT	6.62	6.62	6.62	6.62	6.62	6.62	.00	.00	.00
	GAS	1.20	1.20	1.20	1.20	1.20	1.20	.00	.00	.00
	CHE-O	10.50	9.90	9.00	8.85	9.00	9.00	15.00	15.00	.00
4	CHE-M	17.50	9.90	9.00	8.85	9.00	9.00	25.00	15.00	.00
	CHE-Y	42.00	46.20	42.00	41.30	42.00	42.00	60.00	70.00	100.00
	PCKGE	.24	.26	.24	.24	.24	.24	.34	.40	.57
	MILK	397.34	437.08	397.34	390.72	397.34	397.34	567.63	662.24	946.05
	CREAM	.87	.96	.87	.86	.87	.87	1.24	1.45	2.07
	RENET	1.19	1.31	1.19	1.17	1.19	1.19	1.70	1.99	2.84
5	START	2.79	3.07	2.79	2.74	2.79	2.79	3.98	4.65	6.64
	COLOR	.40	.44	.40	.39	.40	.40	.57	.67	.95
	SALT	1.00	1.10	1.00	.98	1.00	1.00	1.42	1.66	2.37
	LABOR	.08	.09	.08	.08	.08	.08	.12	.14	.20
	ELECT	2.70	2.97	2.70	2.65	2.70	2.70	3.85	4.50	6.42
	GAS	.57	.62	.57	.56	.57	.57	.81	.95	1.35
6	WY-CR	-2.34	-2.57	-2.34	-2.30	-2.34	-2.34	-3.34	-3.90	-5.57
	CN-WY	-38.33	-42.16	-38.33	-37.69	-38.33	-38.33	-54.75	-63.88	-91.25

Figure 4.2. Per 100 lb or percentage basis integrated BOM matrix B_I for the process cheese manufacture

		FD-08	FD-16	PN-08	PN-16	CO-08	CO-16
	FD-08	1.000	.000	.000	.000	.000	.000
	FD-16	.000	1.000	.000	.000	.000	.000
	PN-08	.000	.000	1.000	.000	.000	.000
	PN-16	.000	.000	.000	1.000	.000	.000
	CO-08	.000	.000	.000	.000	1.000	.000
	CO-16	.000	.000	.000	.000	.000	1.000
1	NR-08	.000	.000	.000	.000	.000	.000
	NR-16	.000	.000	.000	.000	.000	.000
	BH-08	.000	.000	.000	.000	.000	.000
	BH-16	.000	.000	.000	.000	.000	.000
	SH-08	.000	.000	.000	.000	.000	.000
	SH-16	.000	.000	.000	.000	.000	.000
	CASE	1.000	1.000	1.000	1.000	1.000	1.000
2	CUP-A	50.000	.000	50.000	.000	50.000	.000
	CUP-B	.000	25.000	.000	25.000	.000	25.000
	CH-FD	25.000	25.000	0.000	0.000	0.000	0.000
	PLN-S	0.000	0.000	25.000	25.000	0.000	0.000
	C&O-S	0.000	0.000	0.000	0.000	25.000	25.000
3	N&R-S	0.000	0.000	0.000	0.000	0.000	0.000
	B&H-S	0.000	0.000	0.000	0.000	0.000	0.000
	S&H-S	0.000	0.000	0.000	0.000	0.000	0.000
4	F-BLN	17.500	17.500	0.000	0.000	0.000	0.000
	S-BLN	0.000	0.000	16.500	16.500	15.000	15.000
	BUTER	0.250	0.250	2.100	2.100	1.500	1.500
	CN-WY	0.000	0.000	2.500	2.500	2.250	2.250
	WPC	2.500	2.500	0.000	0.000	0.000	0.000
	WATER	4.125	4.125	4.625	4.625	4.650	4.650
	EMULS	0.500	0.500	0.500	0.500	0.500	0.500
	SALT	0.125	0.125	0.125	0.125	0.125	0.125
	CHIVE	0.000	0.000	0.000	0.000	0.225	0.225
5	ONION	0.000	0.000	0.000	0.000	0.500	0.500
	RDPEP	0.000	0.000	0.000	0.000	0.000	0.000
	NACHO	0.000	0.000	0.000	0.000	0.000	0.000
	BACON	0.000	0.000	0.000	0.000	0.000	0.000
	HIKOR	0.000	0.000	0.000	0.000	0.000	0.000
	SALAM	0.000	0.000	0.000	0.000	0.000	0.000
	LABOR	0.100	0.100	0.100	0.100	0.100	0.100
	ELECT	1.655	1.655	1.655	1.655	1.655	1.655
	GAS	0.300	0.300	0.300	0.300	0.300	0.300
	CHE-O	2.625	2.625	2.475	2.475	2.250	2.250
6	CHE-M	4.375	4.375	2.475	2.475	2.250	2.250
	CHE-Y	10.500	10.500	11.550	11.550	10.500	10.500

Figure 4.3. Per case basis BOM matrix
for packaged process cheese products

(Figure 4.3 continued)

		NR-08	NR-16	BH-08	BH-16	SH-08	SH-16
	FD-08	.000	.000	.000	.000	.000	.000
	FD-16	.000	.000	.000	.000	.000	.000
	PN-08	.000	.000	.000	.000	.000	.000
	PN-16	.000	.000	.000	.000	.000	.000
	CO-08	.000	.000	.000	.000	.000	.000
	CO-16	.000	.000	.000	.000	.000	.000
1	NR-08	1.000	.000	.000	.000	.000	.000
	NR-16	.000	1.000	.000	.000	.000	.000
	BH-08	.000	.000	1.000	.000	.000	.000
	BH-16	.000	.000	.000	1.000	.000	.000
	SH-08	.000	.000	.000	.000	1.000	.000
	SH-16	.000	.000	.000	.000	.000	1.000
	CASE	1.000	1.000	1.000	1.000	1.000	1.000
2	CUP-A	50.000	.000	50.000	.000	50.000	.000
	CUP-B	.000	25.000	.000	25.000	.000	25.000
	CH-FD	0.000	0.000	0.000	0.000	0.000	0.000
	PLN-S	0.000	0.000	0.000	0.000	0.000	0.000
	C&O-S	0.000	0.000	0.000	0.000	0.000	0.000
3	N&R-S	25.000	25.000	0.000	0.000	0.000	0.000
	B&H-S	0.000	0.000	25.000	25.000	0.000	0.000
	S&H-S	0.000	0.000	0.000	0.000	25.000	25.000
4	F-BLN	0.000	0.000	0.000	0.000	0.000	0.000
	S-BLN	14.750	14.750	15.000	15.000	15.000	15.000
	BUTER	1.450	1.450	1.575	1.575	1.550	1.550
	CN-WY	2.000	2.000	2.250	2.250	2.250	2.250
	WPC	0.000	0.000	0.000	0.000	0.000	0.000
	WATER	4.625	4.625	4.625	4.625	4.625	4.625
	EMULS	0.500	0.500	0.500	0.500	0.500	0.500
	SALT	0.125	0.125	0.100	0.100	0.100	0.100
	CHIVE	0.000	0.000	0.000	0.000	0.000	0.000
5	ONION	0.000	0.000	0.000	0.000	0.000	0.000
	RDPEP	1.150	1.150	0.000	0.000	0.000	0.000
	NACHO	0.400	0.400	0.000	0.000	0.000	0.000
	BACON	0.000	0.000	1.050	1.050	0.000	0.000
	HIKOR	0.000	0.000	0.125	0.125	0.125	0.125
	SALAM	0.000	0.000	0.000	0.000	0.850	0.850
	LABOR	0.100	0.100	0.100	0.100	0.100	0.100
	ELECT	1.655	1.655	1.655	1.655	1.655	1.655
	GAS	0.300	0.300	0.300	0.300	0.300	0.300
	CHE-O	2.213	2.213	2.250	2.250	2.250	2.250
6	CHE-M	2.213	2.213	2.250	2.250	2.250	2.250
	CHE-Y	10.325	10.325	10.500	10.500	10.500	10.500

Figure 4.3. Per case basis BOM matrix for packaged process cheese products

	CH-FD	PLN-S	C&O-S	N&R-S	B&H-S	S&H-S	F-BLN	S-BLN
CH-FD	2000.00	.00	.00	.00	.00	.00	.0	.0
PLN-S	.00	2000.00	.00	.00	.00	.00	.0	.0
C&O-S	.00	.00	2000.00	.00	.00	.00	.0	.0
N&R-S	.00	.00	.00	2000.00	.00	.00	.0	.0
B&H-S	.00	.00	.00	.00	2000.00	.00	.0	.0
S&H-S	.00	.00	.00	.00	.00	2000.00	.0	.0
F-BLN	1400.00	.00	.00	.00	.00	.00	10000.0	.0
S-BLN	.00	1320.00	1200.00	1180.00	1200.00	1200.00	.0	10000.0
BUTER	20.00	168.00	120.00	116.00	126.00	124.00	.0	.0
CN-WY	.00	200.00	180.00	160.00	180.00	180.00	.0	.0
WPC	200.00	.00	.00	.00	.00	.00	.0	.0
WATER	330.00	370.00	372.00	370.00	370.00	370.00	.0	.0
EMULS	40.00	40.00	40.00	40.00	40.00	40.00	.0	.0
SALT	10.00	10.00	10.00	10.00	8.00	8.00	.0	.0
CHIVE	.00	.00	18.00	.00	.00	.00	.0	.0
ONION	.00	.00	40.00	.00	.00	.00	.0	.0
RDPEP	.00	.00	.00	92.00	.00	.00	.0	.0
NACHO	.00	.00	.00	32.00	.00	.00	.0	.0
BACON	.00	.00	.00	.00	84.00	.00	.0	.0
HIKOR	.00	.00	.00	.00	10.00	10.00	.0	.0
SALAM	.00	.00	.00	.00	.00	68.00	.0	.0
LABOR	8.00	8.00	8.00	8.00	8.00	8.00	.0	.0
ELECT	132.40	132.40	132.40	132.40	132.40	132.40	.0	.0
GAS	24.00	24.00	24.00	24.00	24.00	24.00	.0	.0
CHE-O	840.00	924.00	840.00	826.00	840.00	840.00	6000.0	7000.0
CHE-M	350.00	198.00	180.00	177.00	180.00	180.00	2500.0	1500.0
CHE-Y	210.00	198.00	180.00	177.00	180.00	180.00	1500.0	1500.0

Figure 4.4. Process cheese batch formula matrix

Package unit basis BOM matrix

	BLOCK	BAREL
PCKGE	1.0	1.0
BLOCK	40.000	0.00
BAREL	0.000	500.00
MILK	384.528	4730.25
CREAM	0.844	10.35
RENET	1.156	14.20
START	2.696	33.20
COLOR	0.384	4.75
SALT	0.964	11.85
LABOR	0.228	2.85
ELECT	4.268	32.10
GAS	0.548	6.75
WY-CR	-2.264	-27.85
CN-WY	-37.160	-456.25

Batch unit basis BOM matrix

	BLOCK	BAREL
BLOCK	3113.90	0.0
BAREL	0.0	3164.10
MILK	29934.41	29934.41
CREAM	65.59	65.59
RENET	90.00	90.00
START	210.00	210.00
COLOR	30.00	30.00
SALT	45.00	45.00
LABOR	17.70	17.70
ELECT	332.31	203.08
GAS	42.67	42.67
WY-CR	-176.29	-176.29
CN-WY	-2892.67	-2887.23

Figure 4.5. Package unit and batch unit bases BOM matrices for Cheddar cheese manufacture

Table 4.1. Composition and unit cost of the resources available for the process cheese manufacture

Ingredients	Unit	Moisture (%)	Fat (%)	Cost($) per Unit
BUTER	lb	17.0	80.0	1.36
CN-WY	lb	40.0	-	.0178
WPC	lb	1.5	3.5	.480
WATER	lb	100.0	-	.00
EMULS	lb	-	-	.535
SALT	lb	-	-	.120
CHIVE	lb	2.0	-	21.00
ONION	lb	5.0	-	1.37
RDPEP	lb	5.0	15.0	3.95
NACHO	lb	-	-	3.00
BACON	lb	10.0	25.0	4.90
HIKOR	lb	82.0	-	.50
SALAM	lb	35.0	30.0	7.05
LABOR	hour	-	-	10.00
ELECT	KWH	-	-	.065
GAS	therm	-	-	.450
CHE-O	lb	33.0	34.5	1.590
CHE-M	lb	35.0	34.0	1.550
CHE-Y	lb	38.0	33.42	1.1195

	Cost($)/unit	Moisture	Fat
CH-FD	.00	.00	.00
PLN-S	.00	.00	.00
C&O-S	.00	.00	.00
N&R-S	.00	.00	.00
B&H-S	.00	.00	.00
S&H-S	.00	.00	.00
F-BLN	.00	.00	.00
S-BLN	.00	.00	.00
BUTER	1.36	.17	.80
CN-WY	.0178	.40	.00
WPC	.48	.035	.025
WATER	.00	1.00	.00
EMULS	.535	.00	.00
SALT	.12	.00	.00
CHIVE	21.00	.02	.00
ONION	1.37	.05	.00
RDPEP	3.95	.05	.15
NACHO	3.00	.02	.00
BACON	4.90	.10	.25
HIKOR	.50	.82	.00
SALAM	7.05	.35	.30
LABOR	10.00	.00	.00
ELECT	.065	.00	.00
GAS	.45	.00	.00
CHE-O	1.59	.33	.345
CHE-M	1.55	.35	.34
CHE-Y	1.195	.38	.3342

Figure 4.6. A matrix **C** for cost, moisture and fat content of process cheese resources

	CH-FD	PLN-S	C&O-S	N&R-S	B&H-S	S&H-S	F-BLN	S-BLN	CHE-Y
COST($)	100.75	98.49	109.05	108.79	108.64	111.89	133.52	129.94	119.20
H_2O^a(%)	42.57	48.22	45.42	45.03	46.08	46.83	36.50	36.80	38.00
FAT^b(%)	24.66	28.94	25.00	25.19	26.29	26.18	33.73	33.67	33.42

Figure 4.7. A matrix for direct production cost(100 lb basis), moisture and fat contents of products

[a] The moisture content does not include the steam condensate of the cookers which increases the moisture content by 4 to 6 percent.

[ab] Federal Standards of Identity state that process cheese food should contain not more than 44% moisture, and not less than 23% milk fat, while process cheese spreads not less than 44% and not more than 60% moisture, and not less than 20% milk fat.

Bp S =

OCT	NOV	DEC
14140	13840	13751
7140	6960	6889
5461	5270	5144
3019	2970	2856
5100	5120	5022
2020	2080	1938
5500	5400	5710
3300	3160	3570
4112	4050	4025
2448	2430	2295
4800	5300	5234
2960	3420	3326

October	November	December	
14140	13840	13751	FD-08
7140	6960	6889	FD-16
5461	5270	5144	PN-08
3019	2970	2856	PN-16
5100	5120	5022	CO-08
2020	2080	1938	CO-16
5500	5400	5710	NR-08
3300	3160	3570	NR-16
4112	4050	4025	BH-08
2448	2430	2295	BH-16
4800	5300	5234	SH-08
2960	3420	3326	SH-16
60000	60000	59760	CASE
1955650	1949000	1944300	CUP-A
522175	525500	521850	CUP-B
532000	520000	516000	CH-FD
212000	206000	200000	PLN-S
178000	180000	174000	C&O-S
220000	214000	232000	N&R-S
164000	162000	158000	B&H-S
194000	218000	214000	S&H-S
372400	364000	361200	F-BLN
591320	598220	596480	S-BLN
68928	69438	69078	BUTER
87040	88120	87700	CN-WY
53200	52000	51600	WPC
267038	267280	266244	WATER
30000	30000	29880	EMULS
7142	7120	7098	SALT
1602	1620	1566	CHIVE
3560	3600	3480	ONION
10120	9844	10672	RDPEP
3520	3424	3712	NACHO
6888	6804	6636	BACON
1790	1900	1860	HIKOR
6596	7412	7276	SALAM
6000	6000	5976	LABOR
99300	99300	98903	ELECT
18000	18000	17928	GAS
144562	144337	143657	CHE-O
181802	180737	179777	CHE-M
637364	637154	634256	CHE-Y

Y

Figure 4.8. Resource requirements matrix Y over a specific time period

OCT	NOV	DEC	
518	518	519	BLOCK
202	202	201	BAREL
266	260	258	CH-FD
106	103	100	PLN-S
89	90	87	C&O-S
110	107	116	N&R-S
82	81	79	B&H-S
97	109	107	S&H-S

Figure 4.9. Batch requirements matrix over a specific time period

	Matrices:	Original C_0 Cost($)/unit	Revised C_1 Cost($)/unit	Variance C_v Cost($)/unit
	CH-FD	.00	.00	.00
	PLN-S	.00	.00	.00
1	C&O-S	.00	.00	.00
	N&R-S	.00	.00	.00
	B&H-S	.00	.00	.00
	S&H-S	.00	.00	.00
2	F-BLN	.00	.00	.00
	S-BLN	.00	.00	.00
	WY-CR	.0016	.0016	.00
	CN-WY	.0178	.0178	.00
	WPC	.48	.48	.00
	WATER	.00	.00	.00
	EMULS	.535	.535	.00
	SALT	.12	.12	.00
3	CHIVE	21.00	21.00	.00
	ONION	1.37	1.37	.00
	RDPEP	**3.95**	**4.05**	**.10**
	NACHO	3.00	3.00	.00
	BACON	**4.90**	**4.75**	**-.15**
	HIKOR	.50	.50	.00
	SALAM	**7.05**	**7.30**	**.25**
	LABOR	10.00	.00	.00
	ELECT	.065	.00	.00
	GAS	.45	.00	.00
	CHE-O	1.59	1.59	.00
4	CHE-M	1.55	1.55	.00
	CHE-Y	1.195	1.195	.00

Figure 4.10. Original, revised, and variance cost matrices of process cheese resources

CHAPTER 5

BATCHING DECISIONS IN A MULTI-STAGED FOOD MANUFACTURING PROCESS

Abstract

Despite the advance in the process control, batching is a common practice in many process industries for various economic or technological reasons. Production managers often encounter a decision to produce whole or partial batches in the face of variable production targets. Producing whole batches is managerially convenient but may be economically undesirable. Producing partial batches may be managerially and qualitatively inappropriate, but enables achieving an exact production target. Batching decisions directly impact the total volume of final products, resource requirements, and unit costs of products. A model using a penalty approach was constructed to optimize product/batch mix under managerial and manufacturing constraints. The degree of penalty should be determined by the nature of the industry, the type of products, and the conditions of manufacturing and market. The model is applied to an example of production planning for spaghetti sauce products, and is intended as a guide for the construction of similar models in other industries and for other situations.

Introduction

Determining the most suitable number of batches associated with a production target is a complex problem. A batch process occurs when an established quantity of a formula is prepared according to specifications in a single operation. Batching is widely used in the process industries such as food, chemical, petroleum, pharmaceutical, and metal industries (7). Producing batches is part of a manufacturing sequence for intermediate or finished products in a multi-staged process. In the multi-staged process, the batch output from a single batch type (batching device) may be directly or indirectly used to produce several finished products, or several batch types may be used in sequence or simultaneously to produce the finished product(s).

Many food manufacturing systems use batch or semicontinuous processes for various economic or technological reasons. For example, a continuous process is often not appropriate for supporting time-demanding chemical or biological reactions necessary to foster desired quality attributes of products. The batch process offers manufacturing flexibility, which can accommodate modifications of product lines or recipes (86). However, variations in batch yield, production bottlenecks, competition of the products for batch output, and partial/whole batching alternatives complicate the production planning associated with the batch process (82).

In the food industry, finished products are often differentiated by numerous options of flavors, sizes, and packages. The variation in production targets with a discrete production process for multi-staged and multi-products implies difficulties in production planning and decision-making for producing whole or partial batches. Interrelationships among batch types and products complicate product mix and batch mix decisions, which directly impact total volume of output, resource requirements, and unit cost of products. Batch sizes play a significant role in capacity and mix decisions (57). Batch size tends to increase because increasing the batch size not only reduces product unit cost, but also makes labor and process control more efficient than increasing the number of batches. The bigger size may make it more difficult to select partial or whole batches, however.

Whole Batching Versus Partial Batching

Whole batching is preferred for storable products with constant demand, whereas partial batching may be used for products with high production/inventory costs, extreme perishability, and discrete demand. A partial batch is defined as a fraction of a formula for a whole batch. When partial batching is allowed as a production alternative, production managers are faced with a decision to produce partial batches or whole batches of a formula. The batching decision is usually more critical to small food processors which intermittently receive different production orders from various customers, use expensive materials, and are pressed by a low margin and a relatively low volume. The situation is similar to foodservice operations which use a number of batch cookers and change the menu daily so overproduction is costly.

Making whole batches is managerially convenient, but may not be economically desirable. When several types of batches are used to produce the products, producing whole batches for all batch types may be unfavorable. It may not be feasible to produce whole batches when the supply of a specific raw material is restrained due to seasonal fluctuation or increased unit cost of the material. Under certain constraints, making whole or partial batches of product has important consequences that will be explored. When partial and whole batches are compared, consideration should be given to advantages of each. These advantages depend on the type of products and market conditions.

Partial batches generally call for the same equipment time[1] and labor as whole batches, which results in higher unit cost of the products using the batch output. Partial batches may generate variable yields or variable quality attributes. Partial batches, however, can produce the exact volume of output required by a production target. This reduces the costs of overproduction or underproduction.

Both overproduction and underproduction have direct 1 influences on the profitability of the manufacturing. Overproduction generates inventory carrying costs. The inventory carrying costs represent the money invested temporarily in goods for which a company must pay interest on the investment. The inventory carrying costs include opportunity cost, storage and handling cost, taxes, insurance, and shrinking cost for deterioration, obsolescence, pilferage, etc. Food products lose value and may have to be discarded when their shelf lives are reached or the products are damaged or spoiled by undesirable storage conditions such as high temperature, high humidity or insects. Obsolescence happens when inventory cannot be used or sold at full value because of low demand, new

---

[1] The equipment setup time for a partial batch may result in similar or more equipment time as making a whole batch. In general, a partial batch is allocated to the last batch in production.

product development, or formulation modification. Pilferage is theft of inventory by either customers or employees and may be a significant percentage of sales.

Main purposes of keeping inventories are to uncouple bottleneck activities in material flows and to protect against uncertainties in supply, lead time and demand. Many manufacturers whose products have uneven demand rates smooth output rates with inventories. However, keeping large inventories is not desirable for many food processors which manufacture food products with short shelf lives. Food processors often discard the extra batch output, which results in an increase in production costs. In general, build-up of large inventories is avoided in the food industry because of a relatively small time lag between production and consumption and high per unit inventory carrying costs.

Inventory carrying costs are often computed for an item as a percentage of its value, due to a complexity of calculation (49, 72, 61). Stock and Lambert (102) suggested a way to calculate the inventory carrying costs by categorizing the costs into capital costs, inventory service costs, storage space costs, and inventory risk costs. These inventory carrying costs act as pressures for small inventories.

There are also pressures for large inventories, however. These pressures are seasonal raw material supply, customer service, labor and equipment utilization, ordering and setup costs, transportation costs, and purchasing costs. Small inventories may increase the possibility of a stockout or a backorder, and decrease the percentage of on-time deliveries. The seasonality of material supplies or product consumption forces some food processors to store a large amount of input materials or finished products. Labor productivity and equipment utilization can be improved by creating more inventories because the time for machine setup and cleaning decreases, and resource utilization improves. Transportation costs may be lowered because large inventories may allow full carload shipments and minimize the need for expensive, expedited shipments. If a company can tolerate high inventories of raw materials and supplies, it can reduce total purchasing costs by taking advantage of quantity discounts. It is very important to maintain the balance between large inventories and small inventories.

Underproduction may cause loss of profits and future sales potential by not satisfying current sales demand. Customers may be lost due to backordering, particularly if alternative supply sources are available and short lead time is preferred, which is common in the food industry. These cost components of overproduction and underproduction help determine penalties associated with batching decisions.

The batching decision is complicated when there are many intermediate products associated with several batch types. The situation is further complicated when multiple products are produced entirely or partly from the same batch types. In this situation the production manager must know the number of the batches to produce an exact production target and, if it is economically or qualitatively not desirable, decide the most desirable batch mix and product mix. The manager should be also aware of the overproduction and underproduction consequences of his decision. The decision should be objectively driven (i.e., by profit or cost) and be a part of production planning process.

An Example Batch Mix/Product Mix Model

A model optimizing batching decisions is described below by using a penalty approach. Our intent is not to suggest a model which will precisely reflect every circumstance. It is rather to provide an example of a model that may be used to guide in the construction of particular models for specific situations. The values of penalties, the variables and the exclusion or inclusion of constraints must be determined for every different situation. An objective of the model is to find the most desirable product mix and production alternative for these batching decisions. The objective measure will include profits and penalties for the production plan. The penalties will measure losses due to overproduction, underproduction, and partial batching.

Basic Model

Maximize $f(\underline{x},\underline{b})$

$$f(\underline{x},\underline{b}) = \sum_{i\epsilon I} p_i x_i - \{\sum_{i\epsilon I} e_i x_i + \sum_{j\epsilon J} c_j B_j + \sum_{i\epsilon I} z_i (x_i - a_i) t_{1i} +$$

$$\sum_{i\epsilon I} (z_i - 1)(x_i - a_i) t_{2i} + \sum_{j\epsilon J} (B_j - b_j) t_{3j}\} \qquad (5\text{-}1)$$

$$= \sum_{i\epsilon I} \{p_i x_i - e_i x_i - z_i (x_i - a_i) t_{1i} - (z_i - 1)(x_i - a_i) t_{2i}\} -$$

$$\sum_{j\epsilon J} \{c_j B_j - (B_j - b_j) t_{3j}\} \qquad (5\text{-}2)$$

$$\text{or } f(\underline{x},\underline{b}) = (\underline{p}-\underline{e})^t \underline{x} - \underline{c}^t \underline{B} - (\underline{z} \cdot \underline{t}_1)^t (\underline{x}-\underline{a}) -$$

$$\{(\underline{z}-1) \cdot \underline{t}_2\}^t (\underline{x}-\underline{a}) - \underline{t}_3{}^t (\underline{B}-\underline{b}) \qquad (5\text{-}3)$$

Subject to

$$b_j = \sum_{i\epsilon I} r_{ij} x_i, \; j \; \epsilon \; J \qquad (5\text{-}4)$$

$$b_j \le q_j, \; j \; \epsilon \; J \qquad (5\text{-}5)$$

$$\alpha_L a_i \le x_i \le \alpha_U a_i, \; i \; \epsilon \; I \qquad (5\text{-}6)$$

$$B_j = [b_j], \; j \; \epsilon \; J \qquad (5\text{-}7)$$

$$0 \le B_j - b_j \le d_j, \; j \; \epsilon \; J \qquad (5\text{-}8)$$

$$z_i = 1 \text{ if } x_i - a_i \ge 0, \text{ and } z_i = 0, \text{ otherwise}, \quad i \; \epsilon \; I \qquad (5\text{-}9)$$

$$\sum_{i\epsilon I} z_i(x_i - a_i)s_i \le w, \quad i \epsilon I \tag{5-10}$$

where:

I = the index set of finished products with I = {1, 2,---, n};

J = the index set of batch types with J = {1, 2, ---, m};

p_i = selling price per unit of product i, $\underline{p} = [p_1\ p_2 - - p_n]^t$;

e_i = ingredient cost per unit of product i,

$\underline{e} = [e_1\ e_2 - - e_n]^t$;

x_i = number of units of product i, $\underline{x} = [x_1\ x_2 - - - x_n]^t$;

a_i = number of units of product i in a production target,

$\underline{a} = [a_1\ a_2 - - - a_n]^t$;

c_j = labor and utility costs per unit (single occurrence) of batch type j, $\underline{c} = [c_1\ c_2 - - - c_m]^t$;

r_{ij}= Per unit requirements of product i for batch type j

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & - - - - - - & r_{1m} \\ r_{21} & r_{22} & - - - - - - & r_{2m} \\ | & | & & | \\ | & | & & | \\ r_{n1} & r_{n2} & - - - - - - & r_{nm} \end{bmatrix};$$

b_j = number of units of batch type j required for the production of products, $\underline{b}^t = \underline{x}^t\mathbf{R} = [b_1\ b_2 - - - b_m]$;

B_j = the nearest integer not less than b_j(i.e., whole batch corresponding to partial batch b_j), $\underline{B} = [B_1\ B_2 - - - B_n]^t$;

q_j = number of units of batch type j constrained by the ingredient availability or production capacities,

$\underline{q} = [q_1\ q_2\ - - -\ q_m]^t$;

d_j = a minimum limit on a partial batch size of batch type j. The limit is set up high enough to keep economies of batching scale and product quality attributes,

$\underline{d} = [d_1\ d_2\ - - -\ d_m]^t$;

t_{1i}= a penalty for overproduction per unit of product i,

$\underline{t}_1 = [t_{11}\ t_{12}\ - - -\ t_{1n}]^t$;

t_{2i}= a penalty for underproduction per unit of product i,

$\underline{t}_2 = [t_{21}\ t_{22}\ - - -\ t_{2n}]^t$;

t_{3j}= a penalty for partial batches per unit of batch type j,

$\underline{t}_3 = [t_{31}\ t_{32}\ - - -\ t_{3m}]^t$;

α_L = lower limit ratio of acceptable production;

α_U = upper limit ratio of acceptable production;

s_i = space requirement per unit of product i;

w = maximum space allowed for inventories.

The production manager may first examine how many batches for each batch type are required to produce an exact production target. The requirements of the production target for a batch type are computed by multiplying the production target vector $\underline{a}$ by a matrix **R** representing per unit requirements of the products for batch types: $\underline{a}^t\mathbf{R}$. If the batch requirements of the production target include only whole batches or satisfactory, the manager may not have to solve the model. Otherwise, the mixed integer programming model will help the manager find the most suitable product mix and batch mix, which maximize the penaltied profits. The objective function of the model is denoted by $f(\underline{x},\underline{b})$, where vectors $\underline{x}$ and $\underline{b}$ represent a product mix and a batch mix, respectively. In the objective function and constraints, z_i is used to integer variables as needed and other symbols except x_i, b_j, and B_j represent constants. The objective function and constraints of the model can be represented by x_i since the b_j is a function of x_i and r_{ij}, and integer variable B_j is determined by b_j as follows:

$$B_j - b_j = (y_j - 1)b_j = \sum_{i \in I}(y_j - 1)r_{ij}x_i = (y_j - 1)\sum_{i \in I} r_{ij}x_i \qquad (5\text{-}11)$$

$$c_jB_j = c_jy_jb_j = c_jy_j\sum_{i \in I} x_i r_{ij} = \sum_{i \in I} c_j r_{ij} y_j x_i = \sum_{i \in I} u_{ij} y_j x_i$$

$$= y_j \sum_{i \in I} u_{ij} x_i \qquad (5\text{-}12)$$

where

u_{ij} = labor and utility costs from batch type j allocated to one unit of product i at whole batching, $u_{ij} = c_j r_{ij}$;

y_j = proportion of the units of batch type j required by product i in its corresponding whole batch ($y_j \geq 1$)

$$y_j = \frac{B_j}{b_j}.$$

Determination of Unit Production Costs

In the low margin and high volume food industry, it is important to assess the profit margin of a specific product in order to monitor the performance of the product and evaluate its competitive advantage in the market. The batch production of several products, especially when partial batches are made, may result in inaccurate product unit cost and accordingly incorrect product unit profit margin. Labor and utility cost per unit of the product varies depending on the degree of partial or whole batches. When partial batches are made for a batch type, more costs are allocated to the products related with the batch type. The lowest average labor and utility cost per unit of product is therefore attained when whole batches are made for every batch type. To derive an accurate profit margin of a product requires a correct unit production cost of the product.

Unit production cost of the product is determined as follows: The labor and utility cost to meet the total requirement for a specific batch type l is $c_l B_l$. The ratio of the requirement for the number of units of batch type l by a particular product k in the total requirement for batch type l by all products is:

$$\frac{r_{kl}x_k}{b_l}, \tag{5-13}$$

where $b_l = \sum_{i \epsilon I} r_{il}x_i$.

Labor and utility cost for batch type *l* allocated to the production of product *k* is:

$$\frac{(r_{kl}x_k)}{b_l} c_l B_l = (u_{kl}y_k)x_k, \tag{5-14}$$

where $u_{kl} = r_{kl}c_l$ and $y_l = \frac{B_l}{b_l}$.

Thus, production cost for product *k* is:

$$e_k x_k + \sum_{j \epsilon J} u_{kj} y_j x_k, \tag{5-15}$$

and per unit production cost of product *k* is:

$$e_k + \sum_{j \epsilon J} u_{kj} y_j, \tag{5-16}$$

where $u_{kj}y_j$ is average labor and utility cost per unit of product *k*, associated with batch type j and $y_j = 1$ when batch type j is whole batches. Accordingly, total production costs for all products are:

$$\sum_{i \epsilon I} e_i x_i + \sum_{i \epsilon I} \sum_{j \epsilon J} u_{ij} y_j x_i = \sum_{i \epsilon I} e_i x_i + \sum_{j \epsilon J} c_j B_j. \tag{5-17}$$

computing these costs helps assess the effect of the changes in product lines, per batch requirement of a product, or batching costs on the product unit profit margin.

Determination of Penalties

Maximization of projected penaltied profits (profits-minus-penalties) is chosen as an objective function. The maximization function has an advantage of optimal allocation of the multi-product output, which corresponds to ordinary business objectives. We prefer this to a minimization of the sum of production costs and penalties. Both maximization and minimization functions may, however, possess a potential problem determining penalties. Naturally, the profit maximization function increases the production as far as products have positive profit contributions under numerous business constraints, whereas the cost minimization function decreases the production. This problem can be managed by using reasonable degrees of penalties.

The model uses three kinds of penalties:

- penalties for overproduction : $\sum_{i \in I} z_i(x_i-a_i)t_{1i}$;
- penalties for underproduction : $\sum_{i \in I} (z_i-1)(x_i-a_i)t_{2i}$;
- penalties for partial batches: $\sum_{j \in J} (B_j-b_j)t_{3j} = \sum_{j \in J} (y_j-1)b_jt_{3j}$.

These penalties include visible and hidden costs. Visible costs involve inventory costs of storage, obsolescence, perishability, damage, insurance, and tax, costs of sales loss for stockout, and increasing unit product cost by partial batching. Hidden costs involve opportunity cost for the capital required to carry inventories, customer dissatisfaction for stockout or delivery delay, and cost of inefficiency and possible inferior quality for partial batching.

Products have their own specific penalties, depending on the type of products, and the situations of internal and external business environments. For example, overproduction is particularly unfavorable in the production of perishables such as prepackaged and refrigerated foods, products requiring freshness or products requiring expensive materials. For these products, penalties for overproduction must be high. On the other hand, the storable product with low inventory costs and constant demand will have a low penalty for overproduction, and high penalties for underproduction and partial batches. The determinants of penalties are outlined in Exhibit 5.1.

Overproduction is promoted within various business limits when the profit margin for a product is more than the penalty for overproduction of the product. Overproduction does not incur any cost or profit when the unit penalty equals the unit profit margin. In contrast, overproduction incurs costs when the penalty is higher than the profit margin. This option is realistic because overproduction does not generate revenues until it is sold, but incurs inventory carrying costs. Underproduction induces the loss of potential profits due to stockout or delivery delay. Underproduction may not be allowed if backordering or even short supplies significantly has a negative influence on sales or customer satisfaction. Determining a desired customer service level based on customer needs would help measure costs of underproduction. We recognize that excessive overproduction or underproduction may produce different penalties which add nonlinearity to the model.

Partial batching may affect a continuous flow of materials and uniform quality of the products. It is not always desirable to produce whole batches for a variety of managerial reasons such as high inventory costs, backordering costs, insufficient availability of raw materials, inventory control policies (e.g., Just-In-Time production), or distribution constraints. It is especially true when different batch types are used in several intermediate or finished products.

The interrelationships among different batch types and multiple products determine the combination of whole and partial batches for all batch types which is the most desirable. For instance, producing whole batches for a prior batch type may force partial batches for a subsequent batch type. Likewise, producing whole batches for the subsequent batch type may not accommodate the whole batches for the prior batch type. In this situation, different penalties may be given to the batch types according to the batch size, perishability of the batch output, or unit batching costs. Even with partial batches the volume of output may not be exactly the same as that required by the production target. Partial batches produce an intrinsic penalty by incurring higher labor and utility cost per unit of product. Setting minimum levels (lower limits) for partial batches can help reduce the problems of process inefficiency and product quality. The lower limits should consider the resulting quality of the batch output, and the efficiency of managing the partial batches.

The Example

An example illustrates how the model is applied to a production process for spaghetti sauce products. Products in this example are spaghetti sauce and spaghetti sauce with noodles. Units of the products are one case of 12, 32 ounce jars of spaghetti sauce (SS) and one case of 12, 32 ounce jars of spaghetti sauce with noodles (SSN). Batch types associated with the products are batches of spaghetti sauce (SSB) and batches of spaghetti sauce with noodles (SSNB). Thus, spaghetti sauce is used as an intermediate product or as a finished product. Figure 1 outlines a batch production flow of spaghetti sauce products.

The model is applied under the assumptions and procedure described below.

Production target, selling price, and ingredient costs for products

A. A production target is to produce 170 cases of SSN and 160 cases of SS. The production target is organized in a vector $\underline{a}$: $\underline{a} = [a_1\ a_2]^t = [170\ 160]^t$

B. Selling prices per case of products are \$8.59 of SSN and \$5.82 of SS. The selling prices are represented by a vector $\underline{p}$: $\underline{p} = [p_1\ p_2]^t = [8.59\ 5.82]^t$

C. Ingredient costs per case of products are \$5.84 of SSN and \$3.92 of SS. The ingredient costs are represented by a vector $\underline{e}$: $\underline{e} = [e_1\ e_2]^t = [5.84\ 3.92]^t$.

Unit sizes of batch types

Unit batch sizes are 1440 pounds of SSNB and 1500 pounds of SSB. Per batch labor and utility costs are $30.00 of SSNB and $40.00 of SSB. The per batch labor and utility costs represented by a vector $\underline{c}$: $\underline{c} = [c_1\ c_2]^t = [30\ 40]^t$

Per case requirement

A case of SSN requires 24 pounds of spaghetti sauce with noodles, which contains 13.32 pounds of spaghetti sauce. A case of SS requires 24 pounds of spaghetti sauce. Per case requirements for each batch type are:

A. SSN: 24.00/1440 = .017 unit SSNB/case SSN

13.32/1500 = .009 unit SSB/case SSN

B. SS: 0.00/1440 = .000 unit SSNB/case SS

24.00/1500 = .016 unit SSB/case SS

Per case requirements for batch types are organized in a matrix **R**:

R =

	SSNB	SSB
SSN	.017	.009
SS	.000	.016

Per case labor and utility costs

Total labor and utility costs per unit of a product is assumed as the sum of labor and utility costs of the product allocated from each batch type. The lowest average labor and utility cost per unit of product is attained when whole batches are made for both SSNB and SSB. Labor and utility cost for batch type j allocated to one case of product i at whole batching is denoted by u_{ij}, and determined by multiplying per batch labor and utility costs c_j by per case requirements of product i for batch type j r_{ij}:

$$u_{11} = c_1 r_{11} = 30(24.00/1440) = \$.50/\text{case SSN for SSNB}$$
$$u_{12} = c_2 r_{12} = 40(13.32/1500) = \$.36/\text{case SSN for SSB}$$
$$u_{21} = c_1 r_{21} = 30(.00/1440) = \$.00/\text{case SS for SSNB}$$
$$u_{22} = c_2 r_{22} = 40(24.00/1500) = \$.64/\text{case SS for SSB}$$

The lowest labor and utility cost per case of product is $0.86 of SSN and $0.64 of SS. The labor and utility cost for a batch type allocated to one case of a product based on whole batching is organized in a matrix **U**:

U =

	SSNB	SSB	sum
SSN	.50	.36	.86
SS	.00	.64	.64

When partial batches are produced for batch type j, the labor and utility cost per case of product associated with the batch type is determined by multiplying u_{ij} by y_j. For instance, if SSNB is whole batches and SSB is partial batches (4.5 batches), the labor and utility costs per case of SSN and SS are:

$u_{11} + u_{12}(5/4.5) = \$ 0.9$ per case SSN

$u_{21} + u_{22}(5/4.5) = \$ 0.71$ per case SS.

Acceptable production range

Production is assumed acceptable between 97% and 105% of the production target. The supply of resources is assumed limitless.

Per unit penalties

Per unit penalties depend on specific attributes of products. Per unit penalties(t_{1i},t_{2i},t_{3j}) for the production of spaghetti sauce products are determined as follows:

A. Overproduction penalty

It is assumed that SSN and SS have the same storage and ingredient supply conditions. Although the demand for SSN is higher at this period, there has been no apparent preference of consumers for a specific product. Desirability of the penalty higher than the profit margin was previously mentioned. The penalties for overproduction of the products, \$2.73 of SSN and \$2.10 of SS, are assumed as the sum of projected profit margins and inventory-carrying costs for 15 days of average warehousing days. The penalties for overproduction are represented by a vector $\underline{t}_1$:

$$\underline{t}_1 = [t_{11} \; t_{12}]^t = [2.73 \; 2.10]^t.$$

B. Underproduction penalty

The highest possible profit margins are set as penalties for underproduction per unit of products. Variable labor and utility costs incurred by partial batches, may generate different unit profit margins even in the same product at different production periods. The highest profit margin is achieved when whole batches are made for both SSNB and SSB due to their lowest average labor and utility costs per unit of products. The production cost is the sum of material cost, and labor and utility costs. The lowest average production cost per case of product is $6.70 of SSN and $4.56 of SS when whole batches are made for both SSNB and SSB. Accordingly, the highest profit margins or underproduction penalties per case of products are $1.89 of SSN and $1.26 of SS. Higher penalty of SSN implies a potential profit loss will be higher for underproduction of SSN than SS. The penalties for underproduction are represented by a vector $\underline{t}_2$:

$$\underline{t}_2 = [t_{21} \; t_{22}]^t = [1.89 \; 1.26]^t.$$

C. Penalty for partial batches

SSB requires more ingredients, longer blending time, and more careful handling of sensory attributes. Thus, partial batching of SSB needs more attention and a higher penalty is assigned to partial batches of SSB. The penalties for SSNB and SSB are arbitrarily set as 30 and 40, respectively, and represented by a vector $\underline{t}_3$:

$$\underline{t}_3 = [t_{31}\ t_{32}]^t = [30\ 40]^t.$$

Minimum limits on partial batch size is set up high enough to keep economies of batching scale and product quality attributes. The minimum limits for SSNB and SSB are set as 0.5 and 0.7 units of a single batch, respectively. The requirements of the batch type for producing a production target can be determined by multiplying a production target vector $\underline{a}$ by **R**: $\underline{b}^t = \underline{a}^t\mathbf{R} = [170\ 160]\mathbf{R} = [2.89\ 4.09]$. Since 4.09 units of SSB requirement violates the minimum limit, the following model for the production planning of spaghetti sauce is used to obtain the most desirable solution under this specific circumstance.

Maximize $f(\underline{x},\underline{b})$

$$f(\underline{x},\underline{b}) = 2.75x_1 + 1.90x_2 - (30B_1 + 40B_2) - \{2.73(x_1-170)z_1 + 2.10(x_2-160)z_2 + 1.89(x_1-170)(z_1-1) + 1.26(x_2-130)(z_2-1)\} - \{30(B_1-b_1) + 40(B_2-b_2)\} \quad (5\text{-}18)$$

$$= (4.85 - 4.83z_1)x_1 + (3.16 - 3.15z_2)x_2 + 821.1z_1 + 504.0z_2 + 30b_1 - 60B_1 + 40b_2 - 80B_2 - 558.6 \quad (5\text{-}19)$$

subject to

$$(24/1440)x_1 = b_1, \quad (13.32/1500)x_1 + (24/1500)x_2 = b_2 \quad (5\text{-}20)$$

$165 \le x_1 \le 179, \quad 155 \le x_2 \le 168$ (5-21)

$B_1 = [b_1], \quad B_2 = [b_2]$ (5-22)

$.0 \le B_1 - b_1 \le .5, \; .0 \le B_2 - b_2 \le .3$ (5-23)

$z_1 = 1$, if $x_1 \ge 170$, and $z_1 = 0$, otherwise. (5-24)

$z_2 = 1$, if $x_2 \ge 160$, and $z_2 = 0$, otherwise. (5-25)

x_1, x_2, B_1, B_2: nonnegative integers (5-26)

where:

x_i= number of cases of product i (i=1; SSN, i=2; SS);

b_j= number of units of batch type j (j=1; SSNB, j=2; SSB);

B_j= the nearest integer not less than b_j (i.e., whole batch corresponding to partial batch b_j).

Results and Discussion

The problem is solved by using a branch-and-bound method (62, 103). The most desirable solution of the example is (x_1, x_2, b_1, b_2) = (171, 155, 2.85, 4.00), and its objective value is \$501.22. In other words, an actual production optimized is to produce 171 cases of SSN and 155 cases of SS with 2.85 batch units of SSNB and 4 batch units of SSB. In this situation, SSN is overproduced by one case, while SS is underproduced by 5 cases. Total production costs are \$1856.24, and a projected revenue is \$2370.99, which generates \$514.75 as a projected profit. This profit is not equal to the objective value (penaltied profit) because total \$13.53 of penalties are involved for overproduction, underproduction, and partial batches. Table 5.1 summarizes the revenue, costs, profits and penalties involved in this example.

Table 5.2 summarizes the results of three production alternatives: optimal solution (partial batching), whole batching, and producing an exact production target. When partial batching is not allowed in this example, 3 units of SSNB and 5 units of SSB are required to meet the production target. The whole batch production exceeds the production target by 240 pounds of SSN and 1359.6 pounds of SS, which results in a large amount of inventories. Producing an exact production target, on the other hand, requires 2.89 units of SSNB and 4.09 units of SSB, but producing 4.09 units of SSB not only violates minimum limits on partial batch size, but also considerably reduces process efficiency and profitability. It should be noted that the best alternative varies with the production target.

In many circumstances partial batching close to whole batches results in most appropriate production as shown in the example. But whole batching is often preferred due to managerial and technical inconveniences of partial batching despite a possibility of lower profitability. If an inflexible production system or technical problem does not allow partial batches, production plans must be adapted to whole batches in spite of shortages or excess. The basic model can be adjusted to the whole batching policy by placing very high penalties for partial batches, or removing the penalties for partial batches and forcing the batch variables to be integers. Under the whole batching policy, it will be still complicated to solve the batch mix and product mix particularly when there are many intermediates, products, and/or batch types. The adjusted model will certainly help optimizing the batching and product mix decisions.

Exhibit 5.1. Two types of determinants for penalties of products

External determinants

A. market conditions: supplier and buyer markets, competition

B. economic, political and social environments

Internal determinants

A. product types: degree of perishability and obsolescence

B. product life cycle

C. profit contribution margin and market share of product

D. competitive advantage: customer loyalty, product superiority (quality, service, availability, package)

E. resource availability

F. lead time

G. batching cost

H. sensitivity of partial batching to quality attributes of products

I. conditions of inventory and distribution

Table 5.1. Projected revenue, costs, profits, penalties, and penaltied profits of production of spaghetti sauce products

	Unit of Measure	Product SSN	SS	Total
Production target	case	170	160	NM
Actual production	case	171	155	NM
Projected revenue	$	1,468.89	902.10	2,370.99
Ingredient costs	$	998.64	607.60	1,606.24
Labor & util. costs	$	150.80	99.20	250.00
Projected profits	$	319.45	195.30	514.75
Penalties for overproduction	$	2.73	.00	2.73
Penalties for underproduction	$	.00	6.30	6.30
Penalties for partial batches	$	4.50	.00	4.50
Penaltied profits	$	312.22	189.00	501.22

NM: notmeaningful

Table 5.2. Comparison of penaltied profits of three production alternatives

	Optimal solution	Whole batching	Producing a production target
Product mix (SSN/SS cases)	171/155	180/212	170/160
Batch mix (SSNB/SSB units)	2.85/4.0	3/5	2.89/4.09
Projected revenue($)	2,370.99	2780.04	2391.50
Ingredient cost($)	1,606.24	1884.59	1620.00
Labor & util.cost($)	250.00	290.00	290.00
Projected profits($)	514.75	605.45	481.50
Penalties($) for overproduction	2.73	158.76	0
Penalties($) for underproduction	6.30	0	0
Penalties($) for partial batches	4.50	0	39.70
Penaltied profits($)	501.22	446.69	441.80

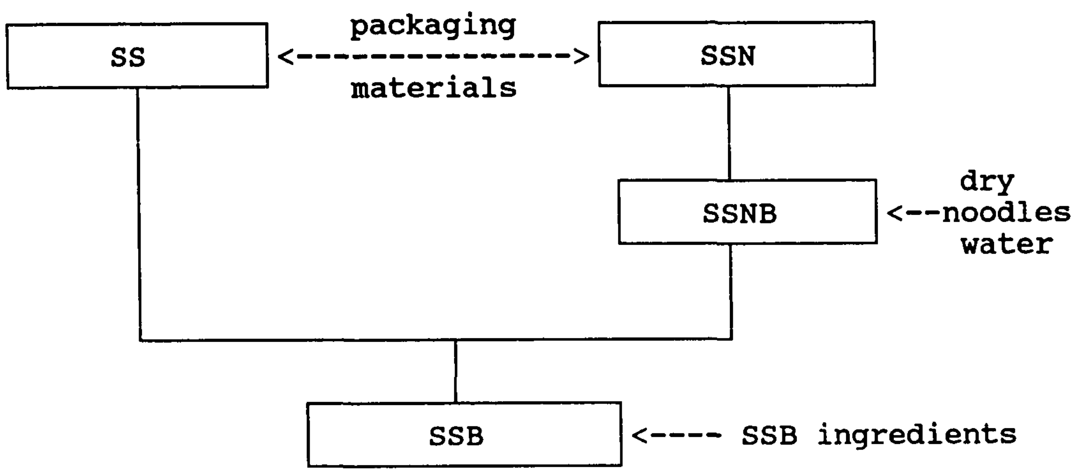


Figure 5.1. Batch production flow of spaghetti sauce products

CHAPTER 6

A ROUTING HEURISTIC AND A CONVEX COMBINATION APPLIED TO A LARGE ROUTING PROBLEM IN FOOD DISTRIBUTION

Abstract

This chapter investigates a large food distributor and describes a heuristic approach for routing (clustering and insertion) procedures and an allocation of drivers and vehicles in food distribution. The heuristic procedures were developed based on the delivery data of 3 days of 4 large geographic regions. The heuristic approach was incorporated to develop an integrated, interactive computer-based system for routing of foodservice delivery vehicles after being tested on the problems of 4 to 5 days of 7 geographic regions which cover the Western, Midwest and Southern United States. The sizes of the problems ranged from 5 to 24 routes per region and 69 to 308 customers per schedule. The revised approach improved the solutions of the previous system by an average of 5.6% of delivery costs. The cost savings were mainly caused by a reduction in the number of routes, which may help the company save fixed costs by reducing the fleet size required as well as variable costs by lowering the number of vehicles.

Foodservice customers may be located on either side of a bay or a river. Convex combinations of delivery points are used to help routing problems associated with a natural boundary such as a bay, a large river, or a mountain range. A cluster first - route second approach assigns deliveries to the routes according to a measure of proximity, and sequences the deliveries on each route. When the natural boundary is not considered, the stops beyond the natural boundary are often assigned to a route with some stops in the depot side due to their proximity. The routing time is therefore underestimated and consequently the routing cost is as well. The measure of proximity without considering the natural boundary often causes erroneous routing schedule in a real distribution situation. A generalized convex combination (weighted average) equation determines whether or not a stop is located beyond a natural boundary. Vehicle routing and scheduling are efficiently managed when the procedure was developed and implemented for the large food distributer.

Introduction

In an economy characterized by high energy costs, rising inflation, potential materials and energy shortages, and declining growth rates in productivity, maintaining a desirable level of corporate profitability is becoming increasingly difficult. The distribution function offers a great potential for profit improvement. In many industries, distribution costs exceed 25 percent of each sales dollar at the manufacturing level (101). Distribution costs are particularly enormous in the food related industries. The U.S. food distribution markets reached $78 billion in 1985 (33). The distribution costs of the soft drink sector comprised about 32 percent of the cost of sales (45). Major distribution costs are driver pay, and vehicle fuel and maintenance costs. Specifically, driver pay accounts for about 35-40 percent of distribution costs.

The foodservice supplier daily delivers to a number of customers small volumes of foodservice products including fresh, perishable and frozen foods. Minimization of total distribution costs by reducing delivery mileage or time as well as the number of routes is a useful goal of vehicle routing. It is especially true with the high-volume and low-margin foodservice supply operation. The competitive position of the foodservice supplier depends on its ability to respond to a large number of frequent or rush orders, and to distribute perishable food products efficiently and reliably. Various managerial constraints complicate vehicle routing decisions. For example, overtime costs caused by reducing the number of routes may lead to more distribution costs, even though total mileage and time are reduced.

Reducing driving distance may not be a good way for minimizing the distribution costs because overtime-related expenses complicate the relation between the distance and overall costs. The routing time involves the driver's settlement, lunch, break, and stop time as well as time for vehicle preparation. In addition, the route may have a different driving speed depending on the geographic situation. These must be considered when distribution costs are determined.

Kraft Inc., a large food distributor, delivers foodservice products to more than 100,000 commercial, institutional and military foodservices in 24 geographic regions in North America. The company has developed the Distribution Decision Support System (DDSS) to make the complicated routing decisions efficiently on a daily basis. The DDSS is an interactive tool which enables a route scheduler to fine tune an initial solution to a time-sensitive routing problem (TSRP) and to deal with last minute changes. The DDSS was based on heuristic approaches proposed by Evans and Norback (33), and a travelling salesman heuristic by Norback and Love (79).

TSRP is defined as a problem in which a fleet of vehicles operating from a single depot are required to distribute products to a number of customers at known locations, where the delivery time is a primary factor in the determination of a complete route (33). Key decisions in TSRP are to determine the delivery time for a particular customer, total routing time, and the balance between reducing the number of routes and overtime costs. Time is a critical factor to the foodservice supplier for efficient and reliable delivery which satisfies the customer.

There has been considerable efforts to develop computer-aided vehicle routing and scheduling systems during 1980s. Availability, users' awareness of potential benefits of using the systems, price drop of software, and advances in computer technologies are major factors for rapid development of computerized vehicle routing and scheduling system (44). Man-machine interactive heuristics coupled with graphic presentation of solutions is suggested as a reasonable method to deal with complex practical problems by combining the human dispatcher's understanding of a problem with the fast computation capability of the computer (92).

In addition, man-machine interactive method offers flexibility in routing and scheduling. There is little known about the vehicle routing algorithms associated with the man-machine interaction and TSRP. A computerized system in which the man-machine interaction and TSRP are applied, controls the food distribution effectively and saved 10.7 percent of overall variable costs for 10 days of deliveries (34).

The goal and basic framework of the vehicle routing problem may be similar, regardless of the characteristics of the industry. But industries or companies within an industry have different distribution policies, regulations, situations, problems, and objectives. This explains why many custom- designed systems have been developed (11, 12, 16, 34, 38, 45, 53). Advances in the technology of food production and preservation as well as changes in the transportation environment have had impact on food distribution, by allowing bigger and more diverse markets, and keener competition. In addition, the foodservice delivery business has the intrinsic nature of low margin and high volume and daily delivery of small volumes of products to a large number of customers. This makes the foodservice vehicle routing problem unique. It is desirable to examine the specific distribution circumstances of the foodservice supplier and develop an algorithm which works well for the practical problems of a significant size, and can be applied to the industry.

Routing Problems

The routing problems Kraft foodservice distribution faced are described below.

Changes in the Food Distribution Environment

Kraft's distribution decision-support system (DDSS) was designed when Kraft foodservice division was much smaller and routing was more easily managed. As Kraft's food distribution network grew, distribution management recognized that the routing procedures of the DDSS did not fit the real distribution situation, due to the expansion of the distribution network and more complicated goals of food distribution.

The competition and low margin of the foodservice supply industry require the company to build a reliable and efficient distribution network system. To achieve such a system, Kraft realized the need to change the goal of the routing procedures of the DDSS. The original goal of Kraft food distribution was to minimize distribution costs. What is new is the addition of constraints to achieve a desired level of customer service such as reliable delivery - timely arrival of quality products during prespecified days or time windows of the customers. For example, customers usually do not want foodservice products to be delivered during the lunch hour when they are very busy. Other crucial constraints are the balance between the supply of distribution resources and the demand of customers, desired route times, the balance of drivers' work loads, the boundary between large cities, and the differentiation between downtown stops and suburban stops in a geographic region.

These changes did not however fit nicely in the clustering approach of the DDSS, which led Kraft to use only a clustering procedure without a subsequent insertion procedure. The DDSS employs a cluster first - route second approach which assigns deliveries to routes, and then sequences the deliveries on each route. While clustering is very complicated due to many constraints and the uncertainty of the optimal number of routes, sequencing is much more manageable because it is essentially a travelling salesman problem.

In many distribution problems the stops are naturally grouped within cities or a certain areas of the suburbs. This is particularly true with foodservice customers. The clustering procedure identifies the natural customer concentrations or clusters, to form the bases for routes. To form a cluster, the original clustering procedure proposed by Evans and Norback (33) employed a heuristic time density function which is defined as a total time estimate for the deliveries contained within a 12 degree cone centered on a specific degree. The clustering procedure computes the time density function for each degree depending on all stops not yet assigned to a route, chooses a 12 degree cone with the highest time density function value and identifies the furthest stop from the depot in the cone. This stop is designated as a seed stop. A cluster centered on the seed stop is then constructed. The size of a cluster is determined by a clustering distance which is calculated by multiplying the straight line distance from the depot and the seed stop by a fixed clustering radius factor (0.5). If a route is incomplete after all stops within the cluster are assigned, the insertion procedure groups isolated stops with the cluster to form a route until any limits of time and capacity are violated. (The clustering and insertion procedures are described in detail with a flow chart in Evans and Norback (33)).

The original insertion procedure determines a stop to be added to a route based on an insertion penalty function

(33). The insertion penalty approach was originally suggested by Fisher and Jaikumar (37). The penalty for the stop k, P_k, is computed according to the formula below.

$$P_k = S_k + R_k - D \qquad (6\text{-}1)$$

where

S_k = the straight line distance between the seed stop and the stop k;

R_k = the straight line distance between the depot and the stop k;

D = the straight line distance between the seed stop and the depot.

As the distribution network expanded, however, it was observed that the insertion penalty approach has a weakness in a certain situation. This situation is depicted in Figure 6.1. In this figure two isolated stops (stops 1 and 2) are outside a cluster. The figure shows that the penalty of stop k equals the difference between the sum of two side line distances (S_k and R_k) and a base line distance (D) of the triangle connecting the seed stop, the depot, and stop k. The penalty function gives the lowest penalty to the stop with the smallest perpendicular line from the base straight line between the depot and the seed stop.

Accordingly, stop 1 has a lower penalty and is chosen as the first stop assigned to the cluster unless any time and capacity limits are violated. After stop 1 is assigned, stop 2 can not fall in the route if any time or capacity limit is violated. This implies that the stop near the cluster may not be assigned to the route and the insertion penalty approach does not guarantee later insertion of the stop near the depot. This results in another long trip to deliver to the isolated stops and more distribution costs.

Kraft accordingly decided temporarily to employ an approach to use the original clustering procedure with the subsequent stop insertion by man-machine interaction using the graphics display of the DDSS. However, the approach was inefficient and time consuming, and tended to make the size of the route too large, and consequently made balancing drivers' work loads difficult. On the other hand, using the clustering procedure without the subsequent insertion procedure resulted in many routes with a small number of stops. The fixed clustering radius factor sometimes fails to keep a route far from the depot from being too large or to keep a route close to the depot from being too small. An appropriate route size depending on the location of the seed stop would improve reliable customer service as well as make a better balance of drivers' work loads.

The time density function does not always guarantee a high density cluster if the center degree measure of the cone is far from the degree measure of the seed stop. A low density cluster is also possible if many stops in the cone are located near the depot or beyond the clustering distance from the seed stop. The approach based on the density function may assign the stop far from the depot late in the process, which can be a single long trip requiring much time and cost. Above all, the expansion of the distribution network and more distribution constraints made it desirable to develop a new routing method which is not only simple, but also easily implemented in the existing DDSS.

Allocating Drivers and Vehicles to Routes

Fleet size is considered a crucial constraint to the vehicle routing problem, but the number of drivers available and their time allowances do not get so much attention yet. Delivery data show the time limits of drivers or company regulation are more constraining than the capacities of vehicles. The capabilities of the vehicles are also important in the distribution of food products since dairy and frozen food products require a vehicle equipped with a refrigeration system. Also, the transportation regulations for the center of a large city may influence the type of the vehicle and the time it may be operated. In addition, the number of vehicles is not usually the same as the number of drivers available. These factors must be considered important limits for creating the route and allocating the driver and vehicle to the route.

A Routing Problem Associated with the Natural Boundary

A special geographic region is defined as a region which contains a natural boundary such as bays, large rivers, mountain ranges, islands, or large lakes. In circumstances where the importance of a reliable customer service is increasing, such natural boundaries have an important impact on vehicle routing and scheduling. If there is a natural boundary between stops assigned to a route, and only a long detour connects the stops, the real routing time will be much longer than the routing time computed by the sequencing procedure. The different measure of proximity for the stops beyond the natural boundary can help reduce the time difference. It does not completely solve the problem, however. The sequencing procedure which uses a travelling salesman algorithm may not produce an optimal delivery schedule by inappropriately sequencing the stops. The resulting routing problem becomes seriously affected when many stops are beyond a natural boundary. A certain natural boundary seems to "cause" clusters of deliveries like restaurants gathered along ocean sides. In this situation the natural boundary may lead to an erroneous routing time estimate and consequently underestimate a route size because the clustering and sequencing procedures estimate the routing time based on the straight distance between the stops without considering the detour. The improper clustering and sequencing negatively influence the optimal allocation of drivers and vehicles according to a route size, drivers' convenient work loads, and routing costs. Clustering and sequencing without regard to natural boundaries can lead to inefficient and unreliable deliveries which do not satisfy distributor or customer requirements.

Kraft recognized the routing problem associated with the natural boundaries as the number and size of geographic regions to be delivered increased. The distributor supposed that vehicle routing and scheduling would be efficiently managed, and thus the customer service level increase if the stops beyond the natural boundary are separately handled from the other stops. The food distributor accordingly differentiated the stops beyond the natural boundary from those on the other side. It was however time-consuming and inefficient to manually identify the stops beyond the boundary, and assign the stops to routes, even with a computer assistance. The difficulty of the manual operation encouraged the need to develop a procedure which manages the natural boundary routing problem and can be easily manipulated in the machine.

To solve the first two problems, a heuristic approach was developed to produce routes, to allocate vehicles and drivers, and to implement an interactive decision-support computer program to make use of these procedures. To solve the last problem, the stops, depot and natural boundary all are assumed to reside in a two dimensional space. A generalized convex combination (weighted average) equation in relation to two dimensional coordinates was developed to determine the location of the stop.

Heuristic Procedures

The heuristic approach was developed by using the data from 3 days of actual deliveries in 4 geographic regions.

Selection and Obtaining Data of Geographic Regions

Each of 24 geographic regions has 3,000 to 5,000 customers in commercial, institutional and military foodservices. The frequent delivery of small volumes of foodservice products may not lead to great daily changes in geographic distribution of deliveries. The delivery data, customer master files, and daily order files for 3 days of 4 geographic regions of which each has a large, unique geographic configuration were obtained to develop this heuristic procedure. A customer master file contains information on each customer such as a customer number, name, and location in terms of X and Y coordinates. A daily order file contains information on customer orders for delivery on a particular date. X and Y coordinates are used to estimate delivery distance and time, and display graphical pictures of the customers and routes. An advantage of using the coordinates is to avoid computer storage for a large interstop distance matrix by computing the distances only when needed and therefore to be able to work with very large problems.

A Revised Clustering Procedure

The revised clustering procedure excludes the concepts of "12 degree cone" and "time density function" of the original clustering procedure (33). In addition, the fixed clustering radius factor is replaced with a variable clustering distance in relation to a straight line distance between the depot and the seed stop. The clustering distance is heuristically determined in a range between 40 and 70 miles, and increases as the seed stop is further from the depot.

A flow chart representation of the clustering procedure is given in Figure 6.2. The following steps depict the procedure: 1) An unassigned stop furthest from the depot, a seed stop, is identified as a starting point to create a route. A cluster is a customer concentration within a certain clustering distance from a seed stop. The advantage of assigning the furthest stop from the depot as a seed stop is that the stops close to the depot can be assigned to almost any route without significantly increasing distribution time and cost. Besides, the route only having stops near the depot can be more easily controlled due to its relatively small size and flexibility such as sending a vehicle without much time and capacity burdens or renting a vehicle for a short time.

2) Among the stops within the clustering distance the unassigned stop closest to the seed stop is the next stop chosen in the attempt to add a new stop.

3) Unless the stop violates any constraint on time and capacities, it is added to the route as a fixed stop. Otherwise, the route is established as a complete route without the stop.

4) The cluster may not have routing time or capacities

enough to be economically acceptable even after all the stops within a clustering distance from the seed stop are assigned. It may then be necessary to add stops to the cluster. The insertion of the stops to the cluster is described in a revised insertion procedure.

A Revised Insertion Procedure

A revised insertion procedure uses the concepts of "variable stops" and a "centre of gravity (CG)". A variable stop is defined as a temporarily assigned delivery that is located within a clustering distance from the CG, the average of X and Y coordinates, of the fixed stops in the cluster. The steps of the insertion procedure represented in Figure 6.2 are depicted in detail as follows:

1) An unassigned stop closest to the CG is sequentially inserted until any constraint is violated. The route with at least one variable stop is labelled an incomplete route after violating any constraint because the variable stops may be reassigned to other incomplete routes by a final insertion criterion.

2) After all stops in a geographic region are assigned to routes, the insertion criterion finally determines the routes of the variable stops by using the CGs associated with the incomplete routes. The criterion first compares

the distance from the variable stop to the CG of its original route with the distances to the CGs of other routes. If no route has shorter distance or the assignment of the stop violates any constraint of the route with shorter distance, then the variable stop remains in the original route. Otherwise, the stop is assigned to the route with the shortest distance. But the comparison of the net distances is not always reasonable. Careful investigation is needed if the variable stop is located further than the seed stop of the candidate route from the depot. The round trip between the variable stop and the candidate route may require more time and mileage than the stop required in the original route. As a rough estimate, this increases the travel distance by twice the distance between the seed stop of the candidate route and the variable stop. Thus, unless the distance between the CG of the original route and the stop is at least two times longer than the distance between the seed stop of the candidate route and the stop, the insertion criterion keeps the stop in the original route. Otherwise, the stop is finally assigned to the candidate route as a new seed stop which is the furthest stop from the depot in the route. The penalty factor protects more costs which can be incurred by assigning a variable stop into the candidate route. The CG of the route is updated whenever a variable stop is assigned to a route by the insertion criterion. In the DDSS the variable stops are shown in a unique color to facilitate the final tuning for the geographic differentiation of the routes.

The desired number of routes is usually set high enough to satisfy all customers. If unassigned stops remain after the number of routes established equals the desired number, the unassigned stop furthest from the depot is assigned to the closest route which can admit the stop in terms of constraints. If the stop may not be inserted into any route, a new route is formed. In many cases these stops are relatively near the depot and do not give much time and capacity burdens to the schedule.

The limit for the minimal number of stops in the last route is removed in the revised approach, because the restriction is especially unreasonable when the stops are scattered and there are enough drivers and vehicles. It is better to have separate routes if the stops do not fall in a route by the clustering approach, not only because the routes near the depot do not usually incur more costs than a large route, but also because the distribution and the customer service are more easily managed.

Allocation of Vehicles and Drivers

Drivers have different driving time allowances, while vehicles have different capacities in terms of weight and volume, and capabilities such as the availability of a refrigerator. When a route is formed, the capacity requirements of the route are related to the capacities and capabilities of vehicles not yet assigned. Similarly, the routing time estimate is related to driving time allowances of various regulations and unassigned drivers. If the trial of adding a stop to a route exceeds the greatest time allowance of the driver or the greatest capacities of the vehicle among those of drivers and vehicles unassigned, the route is established without the stop. After a route is established, the routing time and capacity requirements of the route are compared to the time and capacity limits of drivers and vehicles unassigned, respectively. The route then requires a driver not yet assigned who has the time capacity to do the work. Similarly, an unassigned vehicle must be found which has the capacity to do the deliveries required.

The driver's time limit and vehicle's capacity limit determine the reassignment of variable stops by the final insertion criterion. When the variable stop is reassigned, it is not often hindered because the driver and vehicle of the candidate route usually have enough room for a few additional stops near the route. Moreover, sequencing of stops on each route by a travelling salesman heuristic (32) provides less routing time than the clustering approach. We have found that the route sequence optimization improves the route by a factor of 0.03 approximately. Keeping in mind that actual vehicle performance is stochastic, this factor provides enough "slack" to accommodate the insertion of a few stops in a route. It may be sometimes necessary to exchange drivers or vehicles of the routes due to the addition or deletion of the stops on some routes. If the exchange of drivers or vehicles of the routes is impossible, the variable stop remains as it was. The allocation of drivers and vehicles to the most proper route may lower the number of drivers and vehicles by reducing the number of routes, which results in less distribution costs.

Routing Time and Cost Estimates of the Clustering Approach

The routing time estimate for all stops assigned to a route is expressed as:

$$f(n) = f(n-1) + t(n) = f(1) + \sum_{i=2}^{n} t(i) \qquad (6\text{-}2)$$

This estimate is determined by the following definition of routing time estimates for the first assigned stop (seed stop) and the next stops assigned to the route:

$$f(1=\text{SEED STOP}) = 2kad_1(1)/s + u(1) \quad (6\text{-}3)$$

$$f(i) = f(i-1) + t(i)\ ,\ i = 2,\text{------},n \quad (6\text{-}4)$$

$$t(i) = a\{d_1(i) - d_1(i-1) + d_2(i)\}/s + u(i),\ i = 2,\text{------},n \quad (6\text{-}5)$$

where

2 = the value accounting for the round trip;

k = the transit time factor (1.05-1.20);

a = the factor to approximate the real distance (1.14-1.18);

s = standard driving speed(mph) between the prior stop and the ith assigned stop (30-55);

n = total number of stops assigned to a route;

t(i)= the stop time at the ith assigned stop (The stop time is the sum of the travel time from the prior stop and delivery time at the ith stop.);

u(i)= the delivery time estimate at the ith assigned stop;

$d_1(i)$= the straight line distance between the depot and the ith assigned stop;

$d_2(i)$= the straight line distance between the prior stop and the ith assigned stop.

The transit factor k takes account for the time spent traversing the seed stop on a route (i.e., extra driving time required over the time for a round trip between the depot and the seed stop). This factor decreases as the seed stop is further from the depot (101). The delivery time estimate at a certain stop depends on the type of the loading facility at the stop such as warehouse, military inspection, loading dock, a conveyor, an elevator, etc. The standard driving speed depends on the distance between the stops. The parameters k, a, and s are varied to accommodate different distribution situations of the geographic regions. Finally, the times accounting for driver preparation, settlement, lunch, and break are added to the routing time. A route with a short routing time estimate may not include lunch and break times, however.

The cost estimate for the route j is described as follows:

$$C(j) = C_0 a d(j) + C_1 h_1(j) + C_2 h_2(j) + C_3 X, \ j \in J \qquad (6\text{-}6)$$

Total cost estimate to distribute for the customers on a particular day in a geographic region is therefore:

$$Y = \sum_{j \in 1} C(j) \qquad (6\text{-}7)$$

where

J = the index set of the number of routes in a geographic region, $j \epsilon J$ for j = 1, 2, ------, m;

C_0 = a standard cost per vehicle mile;

a = the factor to approximate the real distance, depending on the geographic region;

d(j)= the straight line distance to travel the route j;

C_1 = an hourly driver pay rate up to 8 regular driving hours per day;

$h_1(j)$= driving hours of regular time at the route j;

C_2 = an overtime driver pay rate per hour;

$h_2(j)$= driving hours of overtime at the route j;

C_3 = overnight expense;

$$X = \begin{cases} 1 & \text{if the route j is an overnight route;} \\ 0 & \text{otherwise.} \end{cases}$$

The Application of Convex Combination (Weighted Average) to Natural Boundary Routing Problem

The location of a delivery point can be expressed in terms of X and Y coordinates in the two dimensional space. The two dimensional coordinates can be used to identify a natural boundary, stops and a depot as well as estimate a driving distance between stops. The geographic distribution of the stops and the routes in the two dimensional space can also be easily displayed in a computerized system. Another advantage of using the two dimensional coordinates is to avoid the computer storage for a large matrix of interstop distances by computing the distance only when needed and therefore to be able to work with large problems.

A convex combination equation of delivery points in relation to the two dimensional coordinates solves the routing problem associated with the natural boundary. Figures 6.3 and 6.4 are presented to show how the equation can be used. Figure 6.3 simply describes a depot and stops with a natural boundary(San Francisco Bay) in San Francisco, California. $\underline{B}_1$ and $\underline{B}_2$ are the points representing the boundary of the bay and will be used to segregate the stops with respect to the natural boundary. By identifying an intersection between a line segment connecting $\underline{B}_1$ and $\underline{B}_2$ and a line segment connecting the depot and a stop, the following convex combination equation determines whether or not a stop is beyond the natural boundary:

$$t_1\underline{B}_1 + (1 - t_1)\underline{B}_2 = t_2\underline{S}_1 + (1 - t_2)\underline{S}_0 \qquad (6\text{-}8)$$

$$\underline{B}_1(X_1,Y_1),\ \underline{B}_2(X_2,Y_2),\ \underline{S}_0(X_0,Y_0),\ \underline{S}_1(X_3,Y_3)\ \epsilon\ E_2 \qquad (6\text{-}9)$$

$$t_1,\ t_2\ \ \epsilon\ [0,1] \qquad (6\text{-}10)$$

where

$\underline{S}_0$ = a depot; $\underline{S}_1$ = a stop;

$\underline{B}_1$, $\underline{B}_2$= points representing a natural boundary.

For each $t_1 \epsilon [0,1]$, any point in the line segment joining $\underline{B}_1$ and $\underline{B}_2$ can be represented by the left-hand side of the equation, and is called a convex combination of $\underline{B}_1$ and $\underline{B}_2$. Similarly, for each $t_2 \epsilon [0,1]$, the right-hand side of the equation can represent any point on the line segment joining the depot and a stop. When the coordinates of the depot, $\underline{S}_0$, are assumed (0,0), the equation (6-8) is expressed as follows:

$$t_1X_1 + (1-t_1)X_2 = t_2X_3 \quad (6\text{-}11)$$

$$t_1Y_1 + (1-t_1)Y_2 = t_2Y_3 \quad (6\text{-}12)$$

The values of t_1 and t_2 can be calculated from the coordinates of $\underline{B}_1$, $\underline{B}_2$, and $\underline{S}_1$:

$$t_1 = (X_3Y_2 - X_2Y_3)/(X_1Y_3 - X_2Y_3 - X_3Y_1 + X_3Y_2) \quad (6\text{-}13)$$

$$t_2 = [t_1X_1 + (1-t_1)X_2]/X_3 \quad (6\text{-}14)$$

If both t_1 and $t_2 \epsilon (0,1)$, then it implies that there is a point of intersection between line segments $\underline{B}_1\underline{B}_2$ and $\underline{S}_0\underline{S}_1$. Hence, $\underline{S}_1$ can be identified as a stop beyond the natural boundary. Unless t_1 and $t_2 \epsilon (0,1)$, on the other hand, there is no intersection point and therefore the stop must be in the depot side. If $t_1 = t_2 = 0$, or $t_1 = 1$ and $t_2 = 0$ in the equations (6-9) and (6-10), the coordinates of the boundary points equal those of the depot since $\underline{B}_2 = (0,0)$ or

$\underline{B}_1$ = (0,0), respectively. If t_1 = 1 and t_2 = 1, or t_1 = 0 and t_2 = 1, the coordinates of the boundary points equal those of the stop beyond the natural boundary because $\underline{B}_1$ = $\underline{S}_1$ and $\underline{B}_2$ = $\underline{S}_1$, respectively. Therefore, the values of t_1 and t_2 of the stops beyond the natural boundary must be more than zero and less than one.

In Figure 6.4, two line segments connecting two dimensional vectors $\underline{B}_j$'s identify the natural boundary. When more than one line segments are needed to identify the natural boundary, the following generalized convex combination (weighted average) equation determines a geographic status of stops:

$$t_1\underline{B}_{2j-1} + (1-t_1)\underline{B}_{2j} = t_2\underline{S}_i + (1-t_2)\underline{S}_0,\ i \in I,\ j \in J \quad (6\text{-}15)$$

$$\underline{B}_j = (X_j,Y_j),\ \underline{S}_0 = (X_0,Y_0),\ \underline{S}_i = (X_i,Y_i),\ i \in I,\ j \in J \quad (6\text{-}16)$$

$$t_1,\ t_2 \in [0,1] \quad (6\text{-}17)$$

where

I = the index set of the number of stops, $i \in I$ for $i = 1,2,----,m$;

J = the index set of the number of line segments identifying a natural boundary, $j \in J$ for $j = 1,2,----,n$;

I and J $\in$ S, S is a nonempty convex set in E_2.

Any point on the line segment joining the two $\underline{B}$ vectors can be described as expressed in the left hand side of the equation (6-15). Similarly, any point on the line segment joining depot $\underline{S}_0$ and stop $\underline{S}_i$ can be described as expressed in the right hand side of the equation (6-15). If the stop is beyond the natural boundary, the line segment joining the depot and the stop must cross at least one line segment identifying the boundary. In other words, the stop must be beyond the boundary if the values of both t_1 and t_2 are between zero and one at least in one equation.

Results and Discussion

We started with the system that requires managers to interact with scheduling and routing procedures. We were asked to automate the system, meet new constraints, and maintain a given level of the customer service satisfaction. Computer codes for the heuristic and convex combination approaches were written in FORTRAN 77 and tested with the sequencing optimization procedures of the existing DDSS at Kraft. With the implementation of revised vehicle routing procedures in the DDSS, the impact of the procedures on the real food distribution situation was determined by testing on the delivery problems of four to five days of seven geographic regions. We accomplished the requests and improved the performance of the system by 5.6 percent. It should be noted that the previous system already improved the delivery performance by 10.7 percent. A comparison of the revised approach to the previous one in six regions is given in Table 6.1, by using the actual delivery problems in DDSS. The problems range from 69 to 308 customers requiring the delivery on one day from a single depot. The numbers given in parenthesis are those resulted from the previous approach. Six regions have four or five days of delivery problems. Corresponding to each day is the number of routes, number of stops, total driving distance, delivery time, and delivery cost estimates. The revised DDSS improved the solutions of the previous one in terms of costs, except two days. Percentage improvements on delivery costs range 1.6 percent to 11.2 percent, averaging 5.6 percent. The improvements were more significant in the regions 1, 3 and 6. More costs of two days of deliveries in the regions 2 and 5 were attributable to overtime routes caused by a constraint on the desired number of routes or many stops within a clustering radius. The delivery costs were affected by the number of routes, overtime routes, total distance, and delivery time. The results show the delivery time is the most significant factor on the delivery costs. At the day 2 of the region 4 the revised approach had more total driving distance, but less delivery time than the previous one. This illustrated that reducing the driving distance is not always the best way for minimizing the delivery costs in the food distribution.

Table 6.2 illustrates significant cost savings by the revised approach, compared to a routing approach using the clustering procedure without the subsequent stop insertion procedure. The region tested was a recently added region, and the sequencing procedure of the previous system was not implemented at the time of testing. As mentioned in the routing problems, the use of the clustering procedure without the subsequent insertion procedure resulted in a large number of routes with a small number of stops.

The cost savings in Tables 6.1 and 6.2 were mainly caused by a reduction of the number of routes required within the constraints of vehicle capacities, drivers' time allowances, balance of drivers' work loads, and desired number of routes. Such a reduction in the number of routes may help the company save fixed costs by reducing the fleet size required or variable costs by lowering the number of vehicles rented.

Table 6.1. Comparison of DDSS results of the previous to the revised approach[a]

Regions	& Days	Number of routes	Number of stops	Distance (miles)	Delivery time(hrs)	Delivery cost($)	Reduction in Cost[b]
1	Day 1	7(8)	113	925(955)	78.2(81.0)	1617(1656)	
	2	9(10)	126	1348(1375)	104.8(105.6)	2269(2280)	
	3	5(6)	74	486(715)	52.4(60.8)	958(1232)	
	4	6(8)	69	582(751)	53.2(61.7)	1030(1229)	
	5	7(7)	81	543(680)	56.9(60.9)	1021(1187)	9.1 %
2	Day 1	20(24)	203	2346(2515)	172.0(182.1)	3669(3895)	
	2	20(27)	287	3236(3499)	215.8(234.0)	4969(5327)	
	3	22(24)	301	2063(2075)	207.0(210.3)	3950(4029)	
	4	23(25)	308	2320(2457)	217.9(225.0)	4211(4448)	
	5	24(28)	307	2703(2521)	232.1(231.7)	4536(4510)	3.9 %
3	Day 1	9(11)	91	948(1061)	73.7(83.3)	1494(1691)	
	2	12(15)	165	736(949)	98.9(110.1)	1618(1945)	
	3	10(13)	152	784(889)	95.9(103.9)	1667(1835)	
	4	8(9)	104	980(1151)	87.1(89.3)	1611(1723)	11.2 %
4	Day 1	11(13)	161	1175(1276)	104.7(110.2)	1767(1867)	
	2	14(15)	173	2270(2263)	146.9(148.2)	2850(2858)	
	3	14(15)	179	2265(2280)	141.2(142.8)	2778(2802)	
	4	14(15)	181	2228(2256)	140.8(142.7)	2764(2792)	1.6 %
5	Day 1	8(8)	89	1087(1106)	79.2(79.7)	2031(2062)	
	2	10(10)	127	1347(1336)	104.9(104.6)	2655(2640)	
	3	7(7)	83	1056(1060)	75.0(75.3)	1956(1968)	
	4	9(9)	137	808(938)	89.7(92.9)	1958(2153)	
	5	11(11)	130	1408(1414)	116.2(116.4)	2878(2883)	2.0 %
6	Day 1	8(11)	101	905(1087)	69.1(77.4)	1817(2109)	
	2	9(13)	88	887(1058)	63.9(77.0)	1726(2062)	
	3	11(14)	168	2238(2476)	125.5(135.8)	4037(4369)	
	4	12(16)	161	2009(2163)	121.0(130.9)	3660(3989)	
	5	10(13)	127	1372(1553)	100.4(105.9)	2750(2971)	9.7 %
Total delivery costs						70347(74512)	
Average improvement(%) of delivery costs per day							5.6 %

[a] Numbers in parentheses indicate the results of the previous approach
[b] Average cost improvement(%) per day of a region of the revised approach over the previous one

Table 6.2. Comparison of DDSS results of a routing approach without an insertion procedure to the revised approach[a]

Region	& Days	Number of routes	Number of stops	Distance (miles)	Delivery time(hrs)	Delivery cost($)
7	Day 1	12(23)	186	2067(4135)	123.4(188.3)	2773(4959)
	2	10(12)	132	1200(1532)	88.6(99.2)	1735(2034)
	3	11(14)	105	1878(3202)	97.0(147.7)	2325(3804)
	4	13(24)	197	2195(4306)	141.4(205.1)	3058(5253)
	5	9(20)	96	1109(2208)	74.5(114.9)	1508(2691)
Total delivery costs						11399(18741)
Average improvement(%) of delivery costs per day						39.2 %

[a] Numbers in parentheses indicate the results of a routing approach without an insertion procedure

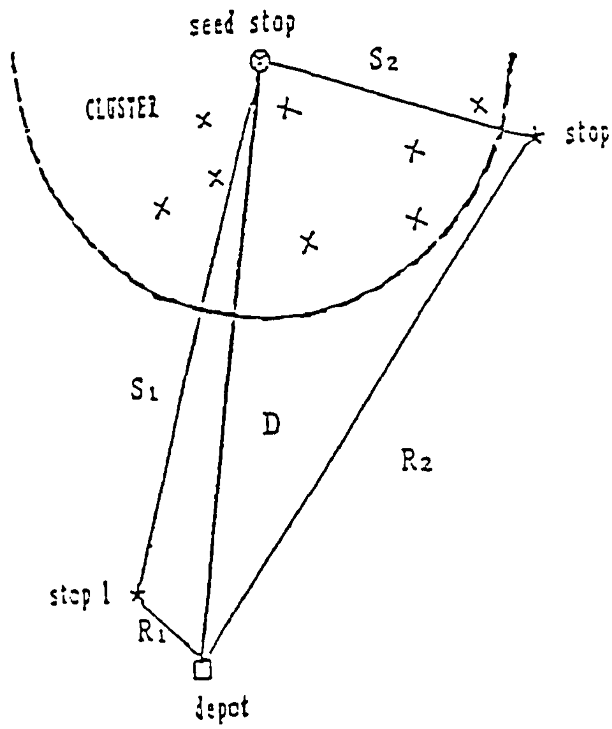


Figure 6.1 An example of the weakness in the insertion penalty approach

$P_1 = S_1 + R_1 - D$, $P_2 = S_2 + R_2 - D$, $P_1 > P_2$

Stop 1 has a lower penalty than stop 2. Thus, stop 1 is chosen as the first stop to be assigned to the route unless any of time and capacity limits is exceeded. Stop 2 may not be added to the route due to a violation of any time and capacity limits. In this case, a long trip to deliver the stop 2 is required and, therefore, more distribution costs are incurred (x : stops in a cluster).

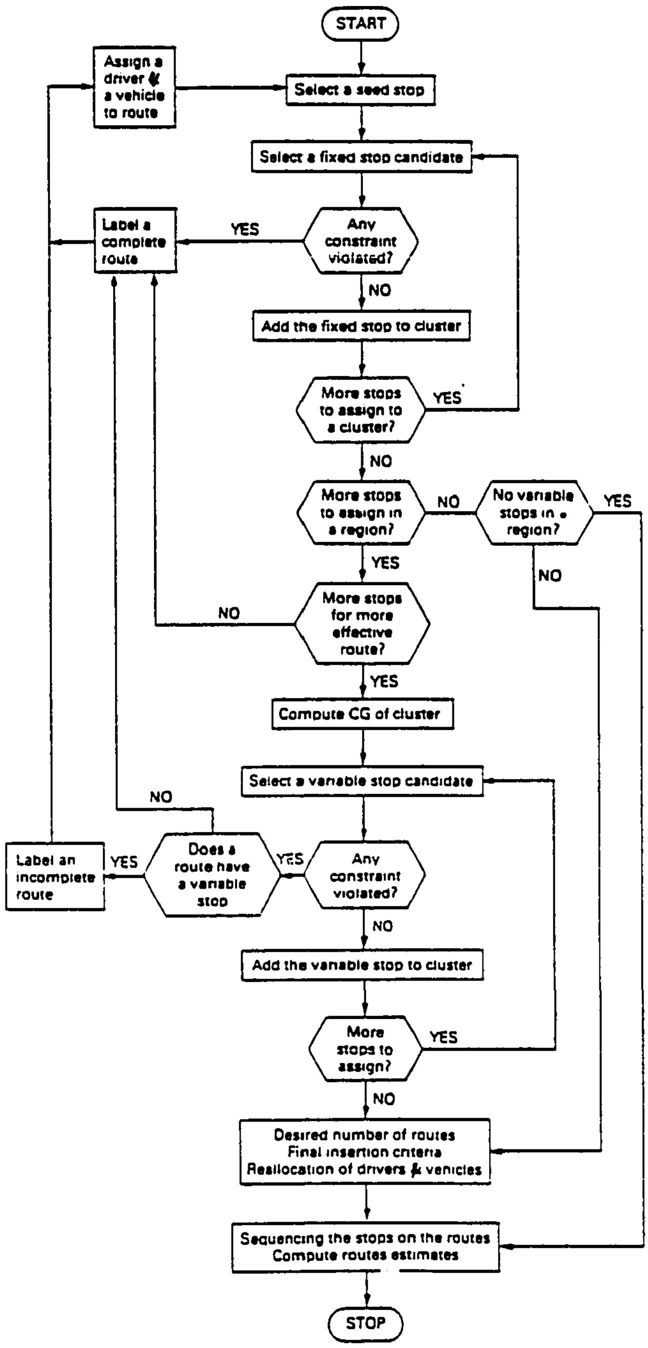


Figure 6.2. Program flow chart

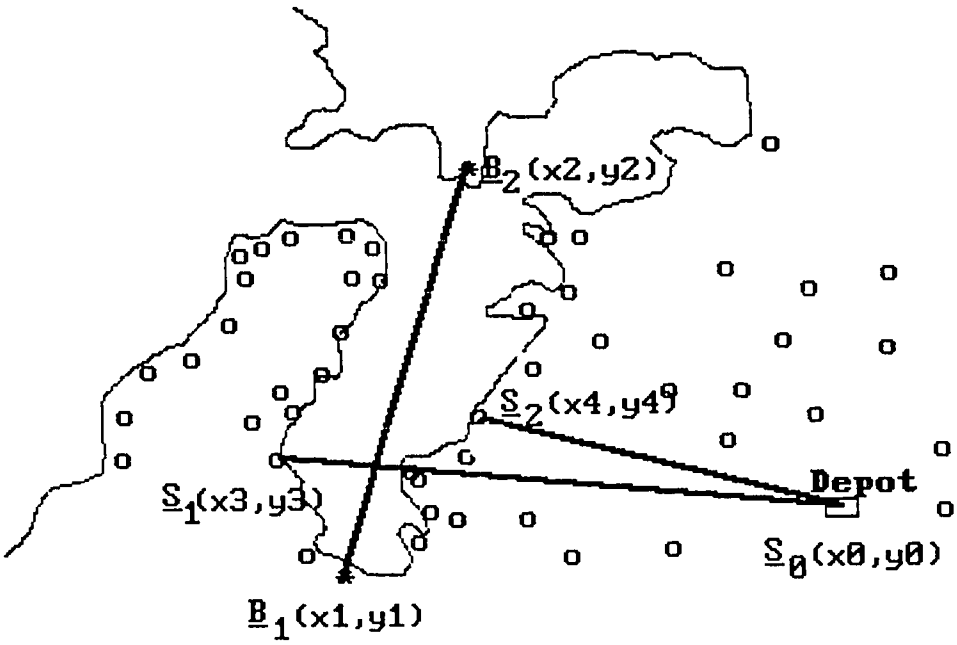


Figure 6.3. A brief representation of a natural boundary (San Francisco bay) routing problem (o: delivery points, *: points representing a natural boundary)

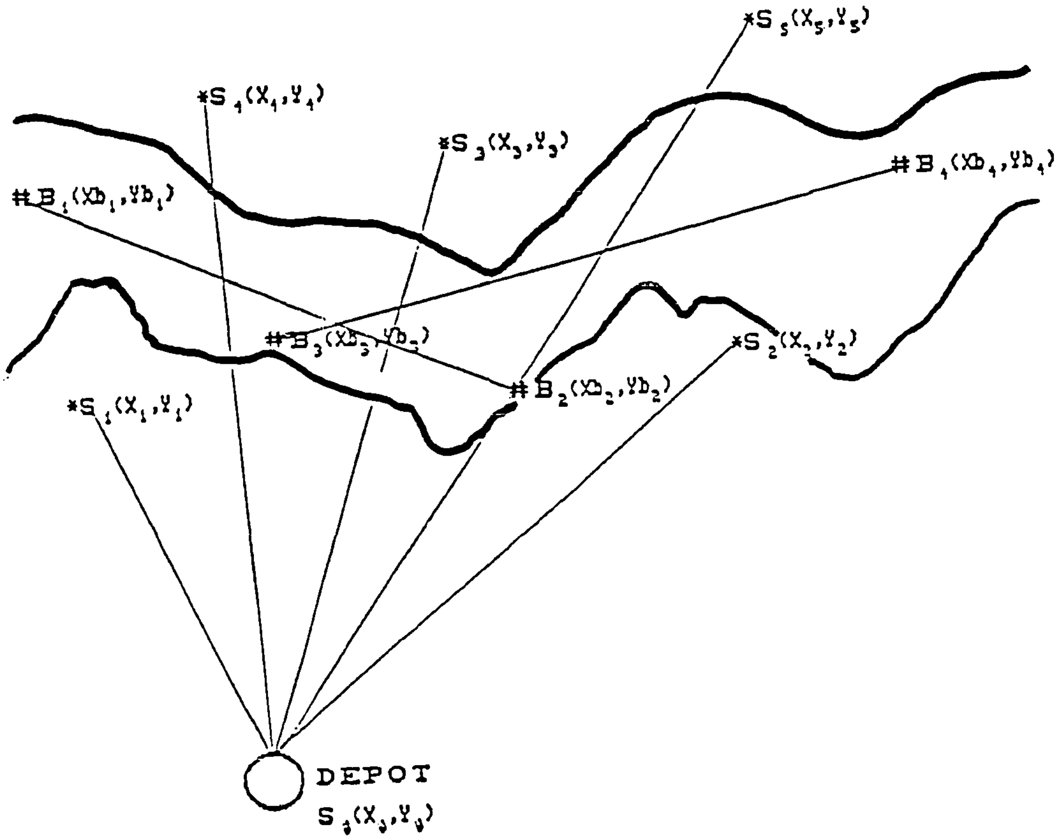


Figure 6.4. A generalized convex combination (weighted average) equation application for identifying a geographic status of stops

$$t_1\underline{B}_1 + (1 - t_1)\underline{B}_2 = t_2\underline{S}_0 + (1 - t_2)\underline{S}_i, \quad i \in I$$
$$t_3\underline{B}_3 + (1 - t_3)\underline{B}_3 = t_2\underline{S}_0 + (1 - t_2)\underline{S}_i, \quad i \in I$$
$$t_1, t_2, t_3 \in [0,1]$$

CHAPTER 7

CONCLUSION AND FUTURE RESEARCH NEEDS

Conclusion

Food processors' unique characteristics and problems relevant to logistics management led us to explore the development of the production planning framework suitable for the food processors. The characteristics and problems include a short lead time, an inverted BOM structure, various measuring units over manufacturing stages, variability in material quality and product yield, batch processes, and a very accurate measurement of resource requirement. High material costs, perishability, and high volume and low profit margins lead food processors to rely on tight logistics management. Food processors' increasing emphasis on profitability rather than sales volume will increase the importance of effective logistics management in the near future.

Mathematical optimization and matrix theory applications offer sound bases for the development of food production planning framework. While mathematical optimization using LP and NLP was used to find satisfactory formulations for the manufacture of Cheddar cheese, NLP and IP were used to solve food production planning problems associated with the batch process. GP provides an integrated bill of materials matrix for multi-stage and multi-product manufacturing, whereas MDS provides a variety of business information supporting decision-making in the food industry settings.

A food processor's business characterized by a high volume and low margin needs to fully utilize information about changing costs and market conditions. A strategy for coping with the changes is to flexibly modify product formulation, product mix, or product price to sustain a desired level of profits. MDS provides a flexible means for managing the changes in data. By incorporating the changes into mathematical optimization and MDS, management can obtain correct information about the impact of the changes on business. For instance, the marketing advantage of being able to switch the product mix in response to demand or cost variances will provide greater incentives for more flexible production planning. The cost savings of flexible product and price management will increasingly exceed manufacturing inconveniences of altering the production plan.

Food processors still commonly use batch processes that produce a predetermined volume of outputs that are used for the manufacture of several products requiring multiple processing stages. Producing an amount exactly equal to a production target may be most desirable, but the actual production may not equal the production target when the batch process is involved. It is especially true when the batch process is associated with the manufacture of several products or more than one batch type is used. These situations make product costing and product/batch mix decision difficult. Chapter five shows how to measure the cost for each product regardless of whole or partial batching practices. A penalty approach is used to support product/batch mix decisions when the partial batching is permitted. When the whole batching is forced or preferred, IP or the penalty model with revised penalty values can be used to optimize the product/batch mix.

Although a vehicle routing procedure introduced in chapter six was developed for a specific food distributor, the procedure may be used for other food distribution or collection problems, where delivery environments are similar.

Future Research Needs

Attempts to reduce the cost of individual activities may lead to increased total costs (or decreased total profits) by causing increases in the costs of other components. For example, cost savings by large volume purchases may be less than the associated increase in inventory carrying costs. Effective management and real cost savings should be accomplished by viewing logistics operations as an integrated system. By integrating the logistics operations, food processors will be able to minimize the total costs of logistics operations from purchasing to distribution rather than minimizing a specific operation cost. The matrix and optimization approaches can be useful tools to help the implementation of the integrated logistics management. The optimization approach can be used to formulate the cost variables of individual logistics operations in a mathematical model to minimize the total costs of the logistics management, while the matrix approach offers a flexible, consistent means of integrating data and obtaining useful business information.

The formulations generated through the mathematical optimization are not necessarily enough to explore the most satisfactory formulations. When the mathematical optimization using LP and NLP was used to find satisfactory formulations for the manufacture of Cheddar cheese, some key quality factors such as salt in moisture (S/M) content and pH are not considered. This is useful since predefined amounts of some ingredients associated with the quality factors like starter cultures, coagulant, and salt provide good barriers against an ineffective formulation optimization. In the optimization of formulations, inclusion of some key quality factors may require nonlinear variables and regression analysis. An evolutionary investigation of best formulations by combining the mathematical optimization and experimentation will be useful to future research.

Chapter four does not present every example which shows how MDS can be applied to the food industry settings. There are and will be more areas that can benefit from MDS applications. For exploring the potential areas, the research must investigate management decisions, the information flow, and managers' information needs.

Matrices logically organize data and MDS provides analytical, structured information that should have meaning to the food processors. With this structure in place, we can identify the needs of integrating the procedures and variables of the optimization and matrix applications into a computerized system. With an assistance of the computer, we can efficiently optimize the entire flow of materials, intermediate products, finished products, manage the information flow of logistics management, and obtain timely managerial decision support information. For building an integrated logistics management system, it would be desirable to involve a database approach as briefly mentioned in chapter one. The database approach results in less redundancy and greater sharing across applications which causes less confusion between organizational units and less time spent resolving errors and inconsistencies in reports. The database approach also permits centralized control over data standards, security restrictions and integrity controls. The matrix form is useful to organize and manipulate data, while it is an integral part of matrix theory applications and can be used to map optimization formulations and solutions into matrices. Another future research need associated with matrix theory applications is to investigate the opportunity to connect GP and material flows with food manufacturing technological facts for quality control purposes, trouble-shooters, and technological evaluations.

More food service customers tend to demand specific desired delivery times of a particular day. Vehicle routing and scheduling problems with the time window constraints were attempted by modifying vehicle schedules of routes, but the approach was not successful when many customers in a route have time windows. This suggests time window constraints would better be solved before a route is formed in the food distribution in which there are frequent daily deliveries to a large number of customers. Several algorithms have been developed but they may not provide complete solutions for the food distribution since they were used in a relatively small number of time constraints, which is different from food distributors' deliveries to a large number of customers.

BIBLIOGRAPHY

1. Anonymous. 1985. Whey used to generate gas for cheese plant. Feedstuff 57(7):47.

2. Anonymous. 1985. Novel devices let firms distribute power beyond traditional boundaries. Computerworld 19(43):81.

3. Anonymous. 1989. State of the food industry-dairy products. Food Engineering 61(6):84.

4. Anonymous. 1989. Computers ease pasta distribution. Transportation and Distribution 30(1): 43-48.

5. Anonymous. 1990. 1990 food industry outlook. Food Processing 51(2):26.

6. Anton, H. 1984. Elementary Linear Algebra. Fourth ed. New York: John Wiley & Sons, Inc.

7. Appoo, P. M. 1987. A practical method of implementing the batch bom in the process industries. Production and Inventory Management 28(2):79-84.

8. Babb, E. M. 1980. Cost and financial performance of Wisconsin cheese plants. Purdue University. Agr. Exp. Sta. Station Bulletin NO. 298.

9. Babb, E. M. and Chambers, J. V. 1988. Economics of dairy processing in the United States. Economics of food processing in the United States edited by McCorkle,Jr. C. O. San Diego, C.A.: Academic Press.

10. Bazaraa, M. S. and C. M. Shetty. 1979. Nonlinear Programming: Theory and Algorithms. New York: Wiley & Sons, Inc.

11. Belardo, S., P. Duchessi and J. Seagle. 1985. Microcomputer graphics in support of vehicle fleet routing. Interfaces 15: 84-92.

12. Bell, W. L. Dalberto, M. Fisher, A. Greenfield, R. Jaikumar, R. Mack and P. Prutzman. 1983. Improving the distribution of industrial gases with an on-line computerized routing and scheduling system. Interfaces 13:4-23.

13. Bender, F.E., A. Kramer and G. Kahan. 1982. Linear programming and its applications in the food industry. Food Technology 36(7):94.

14. Bilkey, M. A. and J. P. Norback. 1986. Using common ingredient units can improve management acceptance of s pport systems for food service operations. Journal of Food Service Systems 6(2):47-58.

15. Brooke, A., D. Kendrick, and A. Meeraus. 1988. GAMS:A User's Guide. Redwood City, C.A.: The Scientific Press.

16. Brown, G. and G. Graves. 1981. Real-time dispatch of petroleum tank trucks. Management Science 27: 19-31.

17. Carne, J. H. 1979. Sliced process cheese production: advantages of a complete system technology, personnel training, engineering and equipment. The 1st Bienniel Marschall International Cheese Conference, Madison, WI. PP 305-308.

18. Cavins, J. F., G. E. Inglett and J.S. Wall. 1972. Linear programming controls amino acid balance in food formulation. Food Technology 26(6): 46.

19. Cegiella, P., W. Szymanowski, and J. Prokuratorski. 1986. COLOS:a collection optimization system as an application of the vehicle scheduling problem. European Journal of Operational Research 27(1): 82-90.

20. Chase, R. D. and N. J. Aquilano 1985. Production and Operations Management. Fourth ed. Homewood, I.L.: Richard D. Irwin, Inc.

21. Cheshire, I. M., A. M. Malleson, and P. F. Naccache. 1982. A dual heuristic for vehicle scheduling. J. Opl. Res. Soc. 33(1): 51-61.

22. Chiang, A. C. 1984. Fundamental of Methods of Mathematical Economics. Third ed. New York: McGraw-Hill.

23. Connor, J. M. 1988. Food Processing: An Industrial Powerhouse in Transition. Lexington, M.S.: Lexington Books.

24. Cook, H. L., Blakely, L., Jacobsen, R. E., Knutson, R. D., Milligan, R., and Strain, J. R. 1978. The dairy subsectors of American agriculture: Organization and vertical coordination. University of Wisconsin, Madison, Wisconsin Ag. Exp. Sta., North Central Regional Res. Pub. 257.

25. Cornell, G. D. 1981. Specifications for procurement of cheese for processing. Paper 37, presented at 2nd Bienniel Marschall International Cheese Conference, Madison, WI.

26. Cox, F., J. Martin and K. Hoffman. 1986. The big rail shippers speak out. Distribution 85(4): 10-24.

27. Craig, K. L., J. P. Norback and M. E. Johnson. 1989. A linear programming model integrating resource allocation and product acceptability for processed cheese products. J. Dairy Sci. 72:3098-3108.

28. Davis, G. M. 1982. Improving work-in-process inventory accuracy in the process industry, Production and Inventory Management 23(2):91-103.

29. Davis, J. E. and J. M. Frank. 1986. Food distribution: the leaders are getting hungry for more. Business Week 2938(March 24, 1986): 106,108

30. Deis, P. 1983. Production and Inventory Management in the Technological Age. Englewood Cliffs, N.J.: Prentice Hall.

31. Evans, J. R. 1986. Spreadsheets and optimization: complementary tools for decision making. Production and Inventory Management 27(4): 36-46.

32. Evans, S. R. 1982. A comprehensive approach to solving the time-sensitive routing problem in a industrial food distribution setting. Ph.D. Thesis, University of Wisconsin, Madison, WI.

33. Evans, S. R. and J. P. Norback. 1984. An heuristic method for solving time-sensitive routing problems. J. Opl. Res. Soc. 35:407-414.

34. ________. 1985. The impact of a decision-support system for vehicle routing in a foodservice supply situation. J. Opl. Res. Soc. 36: 467-472.

35. Fallert, R. F. 1984. Dairy background for 1985 farm legislation. U.S. Department of Agriculture, Agr. Inf. Bul. 474, Washington, D.C.

36. Fisher, M. L. 1981. The lagrangian relaxation method for solving integer programming problems. Management Science 27:1-18.

37. Fisher, M. L. and R. Jaikumar. 1981. A generalized assignment heuristic for vehicle routing. Networks 11:109-124.

38. Fisher, M. L., A. Greenfield, R. Jaikumar and J. Lester. 1982. A computerized vehicle routing application. Interfaces 12:42-52.

39. Fox, M. L. 1987. Integrating logistics systems across functional lines gives companies a competitive edge. Industrial Engineering 19(6): 34-43.

40. Franko, L. G. 1987. New forms of investment in developing countries by US companies: a five industry comparison. Columbia Journal of World Business 27(2): 39-56.

41. Freeman, A., J. Betz, W. Weart, and J. Gordon. 1987. Rail services: casebook. Distribution 86(9): 32-38.

42. Geoffrion, A. M. and R. E. Marsten. 1972. Integer programming algorithms: a framework and state-of-the-art survey. Management Science 18(9):465-491.

43. Golden, B. L., T. L. Magnanti, and H. Q. Nguyen. 1977. Implementing vehicle routing algorithms. Networks 7(1): 113-148.

44. Golden, B. L. and A. A. Assad. 1986. Perspectives on vehicle routing: exciting new developments. Operations Research 34(5): 803-810.

45. Golden, B. L. and E. A. Wasil. 1987. Computerized vehicle routing in the soft drink industry. Operations Research 35: 6-17.

46. Graf, T. F. 1985. National dairy situation-life without goverment support. Presented at 76th American Butter Inst. and National Cheese Inst.

47. Gunn, T. G. 1981. Computer Applications in Manufacturing. New York: Industrial Press, Inc.

48. Heifner, R. 1984. Review of existing and alternative federal dairy programs. U.S. Department of Agriculture, AGES840121, Washington, D.C.

49. Hall, T. W. 1974. Inventory carrying costs: a case study. Management Accounting 7(1):37-39.

50. Hill, B. J. 1971. Some practical implications of mechanization in Cheddar cheese manufacture. Australian J. Dairy Technology 26:132.

51. Hiller, F. S. and Lieberman, G. J. 1986. Introduction to Operations Research. Fourth ed. Oakland, C.A.: Holden-Day, Inc.

52. Horwich, A., Landers, K., Krier, M., and Such, R. 1984. Marketing trends '84. Dairy Record 85(1):52.

53. Hu, P. and M. M. Ahmed. 1989. COMOL: computer-aided operations and management of the lift. Computers and Industrial Engineering 17: 565-569.

54. Hurrion, R. D. 1985. Implementation of a visual interactive consensus decision support system. European Journal of Operational Research 20(2): 138-144.

55. Ishikawa K. 1986. Guide to Quality Control. Second ed. Tokyo: Asian Productivity Organization.

56. Jackson, M. P. and W. G. Wearmouth. 1959. A review of some aspects of processed cheese manufacture. Dairy Science Abstracts 21(5): 177-182.

57. Karmarkar, U. and S. Kekre. 1987. Manufacturing configuration, capacity and mix decisions considering operational costs. Journal of Manufacturing System 6(4):315-324.

58. Kendall, K. E. and M. J. Schniederjans. 1985. Multi-product production planning: a goal programming approach. European Journal of Operational Research 20(1):83-91.

59. Kerrigan, G. L. and J. P. Norback. 1986. Linear programming in the allocation of milk resources for cheese making. J. Dairy Sci. 69:1432.

60. Kosikowski, F., 1982. Cheese and Fermented Milk Foods. Brooktondale, N.Y.: F.V. Kosikowski and Associates.

61. Lambert, D. M. and J. T. Mentzer. 1979. Inventory carrying costs, current availability and uses. International J. Physical Distribution and Materials Management 9(6):256-271.

62. Lawler. E. L. and D. E. Wood. 1966. Branch-and-bound methods: a survey. Operations Research 14(4):699-719.

63. Lawrence, R. C. and J. Gilles. 1980. The assessment of the potential quality of young cheddar cheese. New Zealand J. Dairy Sci. and Technology 15: 1-12.

64. Lawrence, R. C., H. A. Heap, and J. Gilles. 1984. A controlled approach to cheese technology. J. Dairy Sci. 67: 1632-1645.

65. Lelievre, J. 1983. Influence of the casein/fat ratio in milk on the moisture in the non-fat substance in Cheddar cheese. J. Soc. Dairy Technology 36(4): 119.

66. ________. 1983. The influence of cheese composition on the financial return from the manufacture of Cheddar cheese in New Zealand. New Zealand J. of Dairy Sci. and Technology 14: 63-71.

67. Ling, K. C. 1983. Dairy product manufacturing costs at cooperative plants. USDA. Agr. Cooper. Service Research Report No. 34.

68. Lough, H. W. 1975. The cheese industry. Economic Research Service. USDA. Agr. Econ. Rep. No. 294.

69. ________. 1979. Dairy manufacturing capacity and utilization. National economic division, Econ. Stat. and Coop. Service, USDA, Agr. Econ. Rep. No. 427.

70. ________. 1981. Fluid milk processing market structure. U.S. Department of Agriculture. Economic and statistics service, Washington, D.C.

71. Love, R. F., J. G. Morris and G. O. Wesolowsky. 1988. Facilities Location: Models and Methods, New York: North-Holland.

72. Magee, J. F. 1960. The logistics of distribution. Harvard Business Review 38(4):99.

73. Manchester, A. C. 1983. The Public Role in the Dairy Economy. Boulder, C.O.: Westview Press.

74. Martin, J. D. 1986. Investigating shippers' equipment needs (part 2). Distribution 85(1): 10-17.

75. Mesa-Dishington J. K., Aplin, R. D., and Barbano, D. M. 1987. Economic performance of 11 Cheddar cheese manufacturing plants in Northeast and northcentral regions. Agr. Econ. Res. No. 87-2. Dept. of Agr. Econ. Cornell University.

76. Mesa-Dishington J. K., Barbano, D. M., and Aplin, R. D. 1987. Cheddar cheese manufacturing costs economies of size and effects of different current technologies. Agr. Econ. Res. No. 87-3. Dept. of Agr. Econ. Cornell University.

77. Meyer, A. 1973. Processed Cheeese Manufacture. Great Britain: Food Trade Press Ltd.

78. Mize, J.H., C. R. White, and G. H. Brooks. 1971. Operation Planning and Control. Englowood Cliffs, N.J.: Prentice Hall.

79. Norback, J. P. and R. F. Love. 1977. Geometric approaches to solving the traveling salesman problem. Management Science 23: 1208-1223.

80. Norback, J. P. and E. M. Matthews. 1981. Data structures for integrating quality and cost factors in a food service operation. J. Food Protection 44(5):364-368.

81. ________. 1982. Computer implementation of matrix data structures for controlling food formulation and manufacture. Food Technology 36(4):77-82.

82. Norback, J.P. and M. L. Rattunde. 1991. Production planning when batching is part of the manufacturing sequence. J. of Food Process Engineering. (Accepted for publication).

83. Olson, N. F., D. G. Vakaleris, W. V. Price, and S. G. Knight. 1958. Acidity and age of natural cheese as factors affecting the body of pasteurized process cheese spread. J. Dairy Sci 41(8):1005-1015.

84. Olson, N. F. and W. V. Price. 1961. Composition and bacterial growth as factors affecting the body of pasteurized process cheese spread. J. Dairy Sci 44(8): 1394-1404.

85. Olson, N. F. 1974. Mechanized and continous cheesemaking process for Cheddar and other ripened cheese. J. Dairy Sci 58:1015-1021.

86. Overturf, B.W., Reklaitis, G.V. and Woods, J.M. 1978. GASPIV and the simulation of batch/semicontinuous operations: single train process. Ind. Eng. Chem. Process Des. Dev. 17(2):161-165.

87. Pearce, K. N. and J. Gilles. 1979. Composition and grade of cheddar cheese manufactured over three seasons. New Zealand J. Dairy Sci. and Technology 14:63-71.

88. Peters, M. S. and K. D. Timmerhaus. 1980. Plant Design and Economics for Chemical Engineers. Third ed. New York: McGrow-Hill Book Co.

89. Price, W. V. and M. G. Bush. 1974. The process cheese industry in the United States: a review. J. Milk and Food Technology 37(3):135-152.

90. Price, W. V. and M. G. Bush. 1974. The process cheese industry in the United States: a review - II research and development. J. Milk Food Technology 37(4):179.

91. Rice, James W. and John P. Norback. 1987. Process industries production planning using matrix data structures. Production and Inventory Management 28(2): 15-23.

92. Ronen, D. 1988. Perspectives on practical aspects of truck routing and scheduling. European Journal of Operational Research 35(2): 137-145.

93. Salhi, S. and G. K. Rand. 1987. Improvements to vehicle routeing heuristics. J. Opl. Res. Soc. 38(3): 293-295.

94. Salkin, H. M. 1975. Integer Programming. Reading, M.S.: Addison-Wesley Publishing Co.

95. Schill, T. E., J. P. Norback, and E. M. Matthews. 1982. Integrating labor data into a computer model for a foodservice system. Food Technology 36(7):72-75.

96. Scott, R. 1986. Cheesemaking Practice. Second ed. New York: Elsevier Applied Science Publishers.

97. Skinner, R. H. and G. B. Debling. 1969. Food industry applications of linear programming. Food Manufacture 44(10):35.

98. Smith, J.P., B. Ooraikul and E.D. Jackson. 1984. Linear programming: a tool in reformulation studies to extend the shelf life of english-style crumpets. Food Technol. Aust. 36:454.

99. Solomon, M. M. 1987. Algorithms for the vehicle routing and scheduling problems with time window constraints. Operations Research 35(2): 254-265.

100. Stafford, T. H. and Roof, J. B. 1984. Marketing operations of dairy cooperatives. Agricultural Cooperative Service, U.S. Department of Agriculture, Agr. Econ. Rept. 397. Washington, D.C.

101. Stern, L. W. and A. I. El-Ansary. 1988. Marketing Channels. Third ed. Englewood Cliffs, N.J.: Prentice Hall.

102. Stock, J. R. and Lambert, D. M. 1987. Strategic Logistics Management. Second ed. Homewood, IL: Richard D. Irwin, Inc.

103. Taha, H. A. 1975. Integer Programming: Theory, Applications and Computations. New York: Academic Press.

104. ________. 1982. Operations Research. Third ed. New York: Macmillarn Publishing Co. Inc.

105. Tan, C. C. R. amd J. E. Beasley. 1984. A heuristic algorithm for the period vehicle routing problem. Omega 12: 497-504.

106. Templeton, H. L. and H. H. Sommer. 1932. Factors affecting the body and texture of processed cheese. J. Dairy Sci 15(1): 29-41.

107. Thomas, M. A. 1977. The process cheese industry. New South Wales, Australia: Department of Agriculture.

108. Thompson, O. W. and S. J. Connor. 1986. At last: a production planning method tailored to food processors. Food Engineering 58(1):106, 108.

109. ________. 1986. Food process production planning part II-the volume/method matrix. Food Engineering 58(3):196.

110. Tsubone, H., R. Muramatsu, and M. Soshiroda. 1983. Designing an effective production system for processing agricultural products. International Journal of Production Research 21(1):17-30.

111. U.S. Bureau of the Census. 1983. 1982 Census of Manufactures. Washington, D.C.

112. U.S. Bureau of the Census. 1989. 1987 Census of Manufactures. Washington, D.C.

113. ________. 1989. Dairy products annual summary. Statistical Reporting Service, Crop reporting board. Washington, D.C.

114. U.S. Food and Drug Administration 1974. Cheese, Processed Cheese, Cheese Foods and Related Foods. Federal Food, Drug and Cosmetic Act, Part 19, Title 21. Code of Fed. Regul., U.S. Dept. of Health, Education and Welfare, Washington, D.C.

115. Van Slyke, L. L. and W. V. Price. 1979. Cheese. Independence, O.H.: Ridgeview Publishing Co.

116. Vazsonyi, A. 1954. The use of mathematics in production and inventory control. Management Science 1(1):70-85.

117. ________. 1955. The use of mathematics in production and inventory control II. Management Science 1(3-4): 207-223.

118. Werner, F. and R. Manes. 1967. A Standard cost application of matrix algebra. The Accounting Review 41:516-525.

119. Wilster, G. H. 1980. Practical cheesemaking. Corvallis, Oregon: O.S.U. Book Stores, Inc.

120. Yau, C. 1989. A heuristic method for scheduling of trucks from many warehouses to many delivery points. Computers in Industry 11(2): 175-180.

APPENDIX A

GAMS (GENERAL ALGEBRAIC MODELING SYSTEM) MODELS

GAMS (General Algebraic Modeling System) models are formulated to optimize barrel and block Cheddar cheese formulations. GAMS consists of a linear programming solver, a nonlinear programming solver (MINOS), and an integer programming solver (ZOOM). MINOS and ZOOM are optional solvers in GAMS package. The models were solved on a personal IBM-AT compatible computer. The model described in this appendix includes several versions of the model by creating several objective functions. GAMS solved the multiple versions of a model in one job. This appendix contains only models and summary reports of the execution output due to a large volume of original output results. The readers who are interested in GAMS are referred to the user's manual (The Scientific Press, 1989).

Block Cheddar cheese formulation optimization

1) Model

```
$TITLE A MODEL FOR BLOCK CHEDDAR CHEESE FORMULATION OPTIMIZATION

  SETS
    I Output products
      /BLOCK, CRM-REM, WHEYCRM, CONDWHEY/
    J Input Resources
      /MILK, CRM-ADD, NFDM, CONDSKIM/;

  PARAMETERS
    P(I) Unit price of output product i
      /BLOCK      1.3075
       CRM-REM     .8235
       WHEYCRM     .7875
       CONDWHEY    .0780/
    C(J) Unit cost of input resource j
      /MILK        .1197
       CRM-ADD     .8235
       NFDM        .8100
       CONDSKIM    .2390/
    CF1(J) Coefficient of a constraint ensuring min. casein-Fat
       ratio(.68)
      /MILK        .0640
       CRM-ADD  -29.2100
       NFDM      27.3200
       CONDSKIM   8.9484/
    CF2(J) Coefficient of a constraint ensuring max. casein-fat
       ratio(.70)
      /MILK       -.0100
       CRM-ADD  -30.1100
       NFDM      27.3000
       CONDSKIM   8.9410/

*******************************************************************
*  Cheese yield of each input resource is determined by a modified *
*  version of a formula proposed by Van Slyke and Price, and the   *
*  following values:                                               *
*    Moisture = 37%,    Fat retention = .93,                       *
*    Casein retention = .96,   Salt factor = 1.09                  *
*******************************************************************


    CY(J) Cheese yield per 100 pounds of resource j
      /MILK       10.2387
       CRM-ADD    74.7159
       NFDM       48.1157
       CONDSKIM   15.3403/
    WCY(J) Whey cream yield per 100 pounds of resource j
      /MILK        .5736
       CRM-ADD    7.0000
       NFDM        .1556
       CONDSKIM    .0576/ ;
```

```
SCALARS
   CRMCST  Cost of removing cream per lb  /.0016/
   WCRCST  Cost of processing whey cream per lb /.0016/
   CWYCST  Cost of condensing whey per lb  /.0178/
   PKCOST  Packaging material cost per lb block cheese /.015/
   ETCOST  Other direct production costs per vat /378.3/
   CYCRM   Cheese yield of cream removed  /74.8487/
   CFLOCR  Coefficient of cream removed of CF(.68) constraint /29.13/
   CFUPCR  Coefficient of cream removed of CF(.70) constraint /30.03/
   CRLIM   Maximum limit of cream removed from 100 lb milk  /8.22/
   CRWY    Whey cream yield of cream removed /7.00/
   CDWYD   Condensed whey yield per lb separated whey /.1083/;

 PARAMETERS
   SWY(J) Separated yield per 100 pounds of resource j;
     SWY(J) = 100 - (CY(J) + WCY(J));

 VARIABLES
   PROFITS   Total profit contributions from cheesemaking
   REVS      Revenue from cheesemaking and whey processing
   COSTS     Cost of cheesemaking including whey processing
   MAGIN     Profit margin from cheesemaking
   COLB      Cost per lb cheese
   F(I)      Amount of output product i
   X(J)      Amount of input resource j
   SEPWY     Amount of separated whey produced ;

 POSITIVE VARIABLES  F, X;

 EQUATIONS
      PROFIT    Total profit contributions from cheesemaking
      CHCOST    Cheesemaking cost
      CHREV     Cheesemaking revenue
      MARGIN    Profit margin of cheesemaking
      COSTLB    Cost per lb cheese
      VATSIZE   Capacity of a vat(30000 pounds)
      CFLO      A constraint for a minimum casein-fat ratio(.68)
      CFUP      A constraint for a maximum casein-fat ratio(.70)
      CRMLIM    Maximum amount of cream that can be removed
      CHIZYD    Cheese yield per vat
      WCRMYD    Whey cream yield per vat
      SEPWYD    Separated whey yield per vat
      CONDWYD   Condensed whey(60% TS) yield per vat;

    PROFIT..    PROFITS =E= REVS - COSTS;
    CHCOST..    COSTS =E= SUM(J,C(J)*X(J)) + ETCOST +
                    CRMCST*F("CRM-REM") + WCRCST*F("WHEYCRM") +
                    CWYCST*F("CONDWHEY") + PKCOST*F("BLOCK");
    CHREV..     REVS =E= SUM(I,P(I)*F(I));
    MARGIN..    MAGIN*REVS =E= REVS-COSTS;
    COSTLB..    COLB*F("BLOCK") =E= COSTS;
    VATSIZE..   SUM(J, X(J)) - F("CRM-REM") =E= 30000;
    CFLO..      SUM(J, CF1(J)*X(J)) + CFLOCR*F("CRM-REM") =G= 0;
    CFUP..      SUM(J, CF2(J)*X(J)) + CFUPCR*F("CRM-REM") =L= 0;
    CRMLIM..    100*F("CRM-REM") =L= CRLIM*X("MILK");
    CHIZYD..    SUM(J, CY(J)*X(J)) - CYCRM*F("CRM-REM") =E=
                   100*F("BLOCK");
```

```
WCRMYD..    SUM(J, WCY(J)*X(J)) - CRWY*F("CRM-REM") =E=
                100*F("WHEYCRM");

*********************************************************************
* Cheese yield and whey cream yield of cream removed are the same as *
* those of cream removed since their cream percentages are the same: *
*         SWY("CRM-REM") = SWY("CRM-ADD")                            *
* SWY("CRM-REM") is a separated yield per 100 lbs of cream removed.  *
*********************************************************************

SEPWYD..    100*SEPWY =E= SUM(J,SWY(J)*X(J)) -
                SWY("CRM-ADD")*F("CRM-REM");
CONDWYD..   F("CONDWHEY") =E= CDWYD*SEPWY;

MODEL  BKPROFIT /ALL/;
MODEL  BKCOST /ALL/;
MODEL  BKMARGIN /ALL/;
MODEL  BKCOSTLB  /ALL/;

SET K OBJECTIVE MEASURES /REVENUE,COST,PROFIT,MARGIN,COSTLB/ ;

PARAMETER REPORT1(I,*) OUTPUT PRODUCTS SUMMARY REPORT
          REPORT2(J,*) INPUT RESOURCES SUMMARY REPORT
          REPORT3(K,*) ECONOMIC SUMMARY REPORT;

SOLVE BKPROFIT USING NLP MAXIMIZING PROFITS;

REPORT1(I,"PROFIT-MAX") = F.L(I);
REPORT2(J,"PROFIT-MAX") = X.L(J);
REPORT3("REVENUE","PROFIT-MAX") = REVS.L;
REPORT3("COST","PROFIT-MAX") = COSTS.L;
REPORT3("PROFIT","PROFIT-MAX") = PROFITS.L;
REPORT3("MARGIN","PROFIT-MAX") = MAGIN.L;
REPORT3("COSTLB","PROFIT-MAX") = COLB.L;

OPTION LIMROW = 0
OPTION LIMCOL = 0
SOLVE BKCOST USING NLP MINIMIZING COSTS;

REPORT1(I,"COST-MIN") = F.L(I);
REPORT2(J,"COST-MIN") = X.L(J);
REPORT3("REVENUE","COST-MIN") = REVS.L;
REPORT3("COST","COST-MIN") = COSTS.L;
REPORT3("PROFIT","COST-MIN") = PROFITS.L;
REPORT3("MARGIN","COST-MIN") = MAGIN.L;
REPORT3("COSTLB","COST-MIN") = COLB.L;

OPTION LIMROW = 0
OPTION LIMCOL = 0
SOLVE BKMARGIN USING NLP MAXIMIZING MAGIN;

REPORT1(I,"MARGIN-MAX") = F.L(I);
REPORT2(J,"MARGIN-MAX") = X.L(J);
REPORT3("REVENUE","MARGIN-MAX") = REVS.L;
REPORT3("COST","MARGIN-MAX") = COSTS.L;
REPORT3("PROFIT","MARGIN-MAX") = PROFITS.L;
REPORT3("MARGIN","MARGIN-MAX") = MAGIN.L;
```

```
REPORT3("COSTLB","MARGIN-MAX") = COLB.L;

OPTION LIMROW = 0
OPTION LIMCOL = 0
SOLVE BKCOSTLB USING NLP MINIMIZING COLB;
REPORT1(I,"COSTLB-MIN") = F.L(I);
REPORT2(J,"COSTLB-MIN") = X.L(J);
REPORT3("REVENUE","COSTLB-MIN") = REVS.L;
REPORT3("PROFIT","COSTLB-MIN") = PROFITS.L;
REPORT3("COST","COSTLB-MIN") = COSTS.L;
REPORT3("MARGIN","COSTLB-MIN") = MAGIN.L;
REPORT3("COSTLB","COSTLB-MIN") = COLB.L;

DISPLAY REPORT1, REPORT2, REPORT3;
```

2) Solution report summary

```
**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED
                             0     ERRORS
```

---- 167 PARAMETER REPORT1 OUTPUT PRODUCTS SUMMARY REPORT

	PROFIT-MAX	COST-MIN	MARGIN-MAX	COSTLB-MIN
BLOCK	3071.610	3071.610	3113.899	3113.899
WHEYCRM	176.295	172.080	176.295	176.295
CONDWHEY	2892.672	2897.708	2892.672	2892.672

---- 167 PARAMETER REPORT2 INPUT RESOURCES SUMMARY REPORT

	PROFIT-MAX	COST-MIN	MARGIN-MAX	COSTLB-MIN
MILK	29934.413	30000.000	29934.413	29934.413
CRM-ADD	65.587		65.587	65.587

---- 167 PARAMETER REPORT3 BUSINESS SUMMARY REPORT

	PROFIT-MAX	COST-MIN	MARGIN-MAX	COSTLB-MIN
REVENUE	4435.883	4377.664	4435.883	4435.883
COST	4113.940	4067.229	4113.940	4113.940
PROFIT	321.943	310.436	321.943	321.943
MARGIN	0.073	0.071	0.073	0.073
COSTLB	1.321	1.324	1.321	1.321

EXECUTION TIME = 0.115 MINUTES

Barrel Cheddar cheese formulation optimization

1) Model

```
$TITLE A MODEL FOR BARREL CHEDDAR CHEESE FORMULATION OPTIMIZATION

  SETS
    I Output products
      /BARREL, CRM-REM, WHEYCRM, CONDWHEY/
    J Input Resources
      /MILK, CRM-ADD, NFDM, CONDSKIM/ ;

  PARAMETERS
    P(I) Unit selling price of output product i
      /BARREL     1.2650
       CRM-REM     .8235
       WHEYCRM     .7875
       CONDWHEY    .0780/
    C(J) Unit cost of input resource j
      /MILK        .1197
       CRM-ADD     .8235
       NFDM        .8100
       CONDSKIM    .2390/
    CF1(J) Coefficient of a constraint ensuring min. casein-Fat
       ratio(.68)
      /MILK         .0640
       CRM-ADD  -29.2100
       NFDM      27.3200
       CONDSKIM   8.9484/
    CF2(J) Coefficient of a constraint ensuring max. casein-fat
       ratio(.70)
      /MILK        -.0100
       CRM-ADD  -30.1100
       NFDM      27.3000
       CONDSKIM   8.9410/


*******************************************************************
*  Cheese yield of each input resource is determined by a modified *
*  version of a formula proposed by Van Slyke and Price, and the   *
*  following values:                                               *
*    Moisture = 37%,   Fat retention = .93,                        *
*    Casein retention = .96,   Salt factor = 1.09                  *
*******************************************************************


    CY(J) Cheese yield per 100 pounds of resource j
      /MILK       10.4039
       CRM-ADD    75.9210
       NFDM       48.8917
       CONDSKIM   15.8760/
    WCY(J) Whey cream yield per 100 pounds of resource j
      /MILK        .5736
       CRM-ADD    7.0000
       NFDM        .1556
       CONDSKIM    .0576/ ;
```

```
SCALARS
  CRMCST  Cost of removing cream per lb  /.0016/
  WCRCST  Cost of processing whey cream per lb /.0016/
  CWYCST  Cost of condensing whey per lb  /.0178/
  PKCOST  Packaging material cost per lb barrel cheese /.002/
  ETCOST  Other direct production costs  /369.9/
  CYCRM   Cheese yield of cream removed  /76.0560/
  CFLOCR  Coefficient of cream removed of CF(.68) constraint /29.13/
  CFUPCR  Coefficient of cream removed of CF(.70) constraint /30.03/
  CRLIM   Maximum limit of cream removed from 100 lb milk  /8.22/
  CRWY    Whey cream yield of cream removed /7.00/
  CDWYD   Condensed whey yield per lb separated whey  /.1083/;

PARAMETERS
  SWY(J) Separated yield per 100 pounds of resource j;
    SWY(J) = 100 - (CY(J) + WCY(J));

VARIABLES
  COSTS Cost of cheesemaking including whey processing
  COLB  Cost per lb cheesemaking including whey processing
  REVS  Revenue from cheesemaking and whey processing
  F(I)  Amount of output product i
  X(J)  Amount of input resource j
  SEPWY Amount of separated whey produced ;

POSITIVE VARIABLES  F, X;

EQUATIONS
     CHCOST   Cost of making cheese and processing whey
     COSTLB   Cheesemaking cost per lb cheese(objective function)
     VATSIZE  Capacity of a vat(30000 pounds)
     CFLO     Constraint for a minimum casein-fat ratio(.68)
     CFUP     Constraint for a maximum casein-fat ratio(.70)
     CRMLIM   Maximum amount of cream that can be removed
     CHIZYD   Cheese yield per cooking vat
     WCRMYD   Whey cream yield from whey per cooking vat
     SEPWYD   Separated whey yield per cooking vat
     CONDWYD  Condensed whey(60% TS) yield per cooking vat;

  CHCOST..  COSTS =E= SUM(J,C(J)*X(J)) + ETCOST+ CRMCST*F("CRM-REM")
                + WCRCST*F("WHEYCRM") + CWYCST*F("CONDWHEY") +
                PKCOST*F("BARREL");
  COSTLB..   COLB*F("BARREL") =E= COSTS;
  VATSIZE..  SUM(J, X(J)) - F("CRM-REM") =E= 30000;
  CFLO..     SUM(J, CF1(J)*X(J)) + CFLOCR*F("CRM-REM") =G= 0;
  CFUP..     SUM(J, CF2(J)*X(J)) + CFUPCR*F("CRM-REM") =L= 0;
  CRMLIM..   100*F("CRM-REM") =L= CRLIM*X("MILK");
  CHIZYD..   SUM(J, CY(J)*X(J)) - CYCRM*F("CRM-REM") =E=
                100*F("BARREL");
  WCRMYD..   SUM(J, WCY(J)*X(J)) - CRWY*F("CRM-REM") =E=
                100*F("WHEYCRM");

**********************************************************************
* Cheese yield and whey cream yield of cream removed are the same as *
* those of cream removed since their cream percentages are the same: *
*          SWY("CRM-REM") = SWY("CRM-ADD")                           *
* SWY("CRM-REM") is a separated yield per 100 lbs of cream removed.  *
**********************************************************************
```

```
  SEPWYD..    100*SEPWY =E= SUM(J,SWY(J)*X(J)) -
                 SWY("CRM-ADD")*F("CRM-REM");
  CONDWYD..   F("CONDWHEY") =E= CDWYD*SEPWY;

MODEL  BRCOSTLB  /ALL/;
MODEL  BRCOST  /ALL/;

SET K OBJECTIVE MEASURES /COST,COSTLB/ ;

PARAMETER REPORT1(I,*) OUTPUT PRODUCT SUMMARY REPORT
          REPORT2(J,*) INPUT RESOURCES SUMMARY REPORT
          REPORT3(K,*) BUSINESS SUMMARY REPORT;

SOLVE BRCOST USING NLP MINIMIZING COSTS;

REPORT1(I,"COST-MIN") = F.L(I);
REPORT2(J,"COST-MIN") = X.L(J);
REPORT3("COST","COST-MIN") = COSTS.L;
REPORT3("COSTLB","COST-MIN") = COLB.L;

OPTION LIMROW = 0
OPTION LIMCOL = 0
SOLVE BRCOSTLB USING NLP MINIMIZING COLB;

REPORT1(I,"COSTLB-MIN") = F.L(I);
REPORT2(J,"COSTLB-MIN") = X.L(J);
REPORT3("COST","COSTLB-MIN") = COSTS.L;
REPORT3("COSTLB","COSTLB-MIN") = COLB.L;

DISPLAY REPORT1, REPORT2, REPORT3;
```

2) Solution report summary

```
**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED
                             0     ERRORS

----     129 PARAMETER REPORT1       OUTPUT PRODUCT SUMMARY REPORT

            COST-MIN  COSTLB-MIN
BARREL      3121.170    3164.141
WHEYCRM      172.080     176.295
CONDWHEY    2892.341    2887.231

----     129 PARAMETER REPORT2       INPUT RESOURCES SUMMARY REPORT

           COST-MIN  COSTLB-MIN
MILK      30000.000   29934.413
CRM-ADD                  65.587

----     129 PARAMETER REPORT3       BUSINESS SUMMARY REPORT

          COST-MIN  COSTLB-MIN
COST      4018.901    4065.063
COSTLB       1.288       1.285

EXECUTION TIME       =       0.105 MINUTES
```

APPENDIX B

DERIVATION OF THE GOZINTO PROCEDURE

The integrated process cheese manufacturing system described in the gozinto matrix **T** can be viewed as a three-level system as follows:

1. The first level is to cook cheese blend and other direct ingredients to manufacture process cheese products.
2. The second level is to make cheese blend by mixing young, medium and old aged Cheddar cheeses.
3. The third level is to make young Cheddar cheese by using its direct ingredients.

To explore how GP is derived, the recipe matrix **R** is partitioned as follows:

$$\mathbf{R} = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{cccc} I & II & III & IV \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{R}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R}_3 & \mathbf{0} \end{array}\begin{array}{c} \\ I \\ II \\ III \\ IV \end{array}$$

where:

I = process cheese products - CHEESE FOOD, PLAIN SPREAD, CH&ON SPREAD, NC&RP SPREAD, BC&HI SPREAD, SL&HI SPREAD

II = direct ingredients of process cheese products

F-BLN, S-BLN, WY-CR, CN-WY, WPC, WATER, EMULS, SALT, CHIVE, ONI-F, R-PEP, NACHO, BACON, HIKOR, SALAM

III = direct ingredients for cheese blend

CHE-O, CHE-M, CHE-Y

IV = direct ingredients and byproducts of young Cheddar cheese

MILK, CREAM, RENET, START, COLOR, SALT, WY-CR, CN-WY

$\mathbf{R}_1$ = a submatrix that describes the relationship between I and II. This matrix describes the first level (manufacturing process cheese) of the process cheese manufacturing system.

$\mathbf{R}_2$ = a submatrix that describes the relationship between II and III. This matrix describes the second level (making cheese blend) of the process cheese manufacturing system.

$\mathbf{R}_3$ = a submatrix that describes the relationship between III and IV. This matrix describes the third level (manufacturing Cheddar cheese) of the process cheese manufacturing system.

The submatrices R_1, R_2, and R_3 are described as follows:

R_1 =

CHEESE FOOD	PLAIN SPREAD	CH&ON SPREAD	NC&RP SPREAD	BC&HI SPREAD	SL&HI SPREAD	
.700	0	0	0	0	0	F-BLN
0	.66	.60	.59	.60	.60	S-BLN
.010	.084	.060	.058	.063	.062	BUTER
0	.10	.09	.08	.09	.09	CN-WY
.100	0	0	0	0	0	WPC
.165	.185	.186	.185	.185	.180	WATER
.020	.020	.020	.020	.020	.020	EMULS
.005	.005	.005	.005	.004	.004	SALT
0	0	.0090	0	0	0	CHIVE
0	0	.0200	0	0	0	ONI-F
0	0	0	.0046	0	0	R-PEP
0	0	0	.0160	0	0	NACHO
0	0	0	0	.0420	0	BACON
0	0	0	0	.0050	.0050	HIKOR
0	0	0	0	0	.0420	SALAM
.004	.004	.004	.004	.004	.004	LABOR
.0662	.0662	.0662	.0662	.0662	.0662	ELECT
.012	.012	.012	.012	.012	.012	GAS

R_2 =

F-BLN	S-BLN	BUTER	-	-	-	-	-	-	-	-	-	-	-	GAS	
.15	.15	0	0	0	0	0	0	0	0	0	0	0	0	0	CHE-O
.25	.15	0	0	0	0	0	0	0	0	0	0	0	0	0	CHE-M
.60	.70	0	0	0	0	0	0	0	0	0	0	0	0	0	CHE-Y

R_3 =

CHE-O	CHE-M	CHE-Y	
0	0	.002	PKAGE
0	0	9.4605	MILK
0	0	.0207	CREAM
0	0	.0284	RENET
0	0	.0664	START
0	0	.0095	COLOR
0	0	.0237	SALT
0	0	.0057	LABOR
0	0	.0642	ELECT
0	0	.0135	GAS
0	0	-.0557	WY-CR
0	0	-.9125	CN-WY

R_3 is multiplied by R_2:

R_3R_2 =

F-BLN	S-BLN	WY-CR	-	-	-	-	-	-	-	-	-	-	-	SALAM	
.0034	.0040	0	0	0	0	0	0	0	0	0	0	0	0	0	PKAGE
5.6763	6.6224	0	0	0	0	0	0	0	0	0	0	0	0	0	MILK
.0124	.0145	0	0	0	0	0	0	0	0	0	0	0	0	0	CREAM
.0170	.0199	0	0	0	0	0	0	0	0	0	0	0	0	0	RENET
.0398	.0465	0	0	0	0	0	0	0	0	0	0	0	0	0	START
.0057	.0067	0	0	0	0	0	0	0	0	0	0	0	0	0	COLOR
.0142	.0166	0	0	0	0	0	0	0	0	0	0	0	0	0	SALT
.0012	.0014	0	0	0	0	0	0	0	0	0	0	0	0	0	LABOR
.0385	.0450	0	0	0	0	0	0	0	0	0	0	0	0	0	ELECT
.0081	.0095	0	0	0	0	0	0	0	0	0	0	0	0	0	GAS
-.0334	-.0390	0	0	0	0	0	0	0	0	0	0	0	0	0	WY-CR
-.5475	-.6388	0	0	0	0	0	0	0	0	0	0	0	0	0	CN-WY

The resulting matrix organizes per lb direct ingredient requirement of process cheese products (II) for young Cheddar ingredients (IV). This matrix is exactly matched with a sub matrix that describes the relationship between II and IV in the matrix **T**.

R_2 is multiplied by R_1 to obtain the product (I) requirement for cheese blend ingredients (III):

R_2R_1=

CHEESE FOOD	PLAIN SPREAD	CH&ON SPREAD	NC&RP SPREAD	BC&HI SPREAD	SL&HI SPREAD	
.105	.099	.009	.0885	.090	.090	CHE-O
.175	.099	.009	.0885	.090	.090	CHE-M
.420	.462	.420	.413	.420	.420	CHE-Y

The resulting matrix is exactly the same as the submatrix describing the relationship between I and II in the matrix **T** (per lb basis).

Finally, to derive the product (I) requirement for young Cheddar cheese ingredients (IV), R_3 is multiplied by R_2R_1:

$R_3(R_2R_1)$ =

CHEESE FOOD	PLAIN SPREAD	CH&ON SPREAD	NC&RP SPREAD	BC&HI SPREAD	SL&HI SPREAD	
.0024	.0026	.0024	.0024	.0024	.0024	PKAGE
3.9734	4.3708	3.9734	3.9072	3.9734	3.9734	MILK
.0087	.0096	.0087	.0086	.0087	.0087	CREAM
.0119	.0131	.0119	.0117	.0119	.0119	RENET
.0279	.0307	.0279	.0274	.0279	.0279	START
.0040	.0044	.0040	.0039	.0040	.0040	COLOR
.0100	.0110	.0100	.0098	.0100	.0100	SALT
.0008	.0009	.0008	.0008	.0008	.0008	LABOR
.0270	.0297	.0270	.0265	.0270	.0270	ELECT
.0057	.0062	.0057	.0056	.0057	.0057	GAS
-.0234	-.0257	-.0234	-.0230	-.0234	-.0234	WY-CR
-.3833	-.4216	-.3833	-.3769	-.3833	-.3833	CN-WY

The resulting matrix is the same as the submatrix that shows the relationship between I and IV in the matrix **T**. Therefore, gozinto matrix **T** can be represented by the sum of the identity matrix with the same size as **T** and the aggregate matrix **A** in the following way:

$$\mathbf{T} = \mathbf{I} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ R_1 & 0 & 0 & 0 \\ R_2R_1 & R_2 & 0 & 0 \\ R_3R_2 & R_1R_3R_2 & R_3 & 0 \end{bmatrix} = \mathbf{I} + \mathbf{A} \qquad (1)$$

The aggregate matrix **A** organizes both direct and indirect relationships among products, intermediate products, and ingredients. To derive the relationships between **T** and **R**, $\mathbf{T} = (\mathbf{I} - \mathbf{R})^{-1}$, the following procedure is described:

1. Multiply **R** by **R**.

$$\mathbf{R}^2 = \mathbf{RR} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ R_2R_1 & 0 & 0 & 0 \\ 0 & R_3R_2 & 0 & 0 \end{bmatrix} \qquad (2)$$

2. Multiply **R** by $\mathbf{R}^2$.

$$\mathbf{R}^3 = \mathbf{R}\mathbf{R}^2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ R_3R_2R_1 & 0 & 0 & 0 \end{bmatrix} \quad (3)$$

3. Multiply **R** by R_3: $\mathbf{R}^4 = \mathbf{R}\mathbf{R}^3 = 0$. (4)

$\mathbf{R}^4$ is a null matrix and, consequently, $\mathbf{R}^5$ -----,$\mathbf{R}^m$ are null matrices. This shows that 4 or more factors of the square matrix **R** is zero matrices when a matrix is a 3-level, lower triangular matrix with zero entries along the main diagonal. Now, it can be generalized that $\mathbf{R}^{m+1}$ is a null matrix when the manufacturing system is m-level.

4. Add **R**, $\mathbf{R}^2$, and $\mathbf{R}^3$.

$$\mathbf{R} + \mathbf{R}^2 + \mathbf{R}^3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ R_1 & 0 & 0 & 0 \\ R_2R_1 & R_2 & 0 & 0 \\ R_3R_2R_1 & R_3R_2 & R_3 & 0 \end{bmatrix} = \mathbf{A} \quad (5)$$

5. Add **A** to the itentity matrix **I** with the same size as **A**.

$\mathbf{A} + \mathbf{I} = \mathbf{T}$ (6)

Thus, **T** can be expressed as follows:

$\mathbf{T} = \mathbf{I} + \mathbf{A}$

$= \mathbf{I} + \mathbf{R} + \mathbf{R}^2 + \mathbf{R}^3$ (7)

6. Finally, Gozinto Procedure, $\mathbf{T} = (\mathbf{I} - \mathbf{R})^{-1}$, is derived by multiplying (I-R) in both sides of the equation (7) on the left.

$$
\begin{aligned}
(\mathbf{I} - \mathbf{R})\mathbf{T} &= (\mathbf{I} - \mathbf{R})(\mathbf{I} + \mathbf{R} + \mathbf{R}^2 + \mathbf{R}^3) \\
&= \mathbf{I}^2 + \mathbf{R} + \mathbf{R}^2 + \mathbf{R}^3 - \mathbf{R} - \mathbf{R}^2 - \mathbf{R}^3 - \mathbf{R}^4 \\
&= \mathbf{I} - \mathbf{R}^4 \\
&= \mathbf{I} \qquad (8)
\end{aligned}
$$

The same result occurs when **T** is multiplied both sides of the equation (7) on the right by (I - R).

$$\mathbf{T}(\mathbf{I} - \mathbf{R}) = (\mathbf{I} + \mathbf{R} + \mathbf{R}^2 + \mathbf{R}^3)(\mathbf{I} - \mathbf{R}) = \mathbf{I}. \qquad (9)$$

The equations (8) and (9) indicate that (I - R) and **T** are invertible and are inverses to each other. Similarly, Gozinto Procedure, $\mathbf{T} = (\mathbf{I} - \mathbf{R})^{-1}$, will also be attained when the manufacturing system is m-level:

$$
\begin{aligned}
(\mathbf{I} - \mathbf{R})\mathbf{T} &= (\mathbf{I} - \mathbf{R})(\mathbf{I} + \mathbf{R} + \mathbf{R}^2 + \mathbf{R}^3 + - - - + \mathbf{R}^m) \\
&= \mathbf{I} - \mathbf{R}^{m+1} \\
&= \mathbf{I} \qquad (10)
\end{aligned}
$$

From (9) and (10), therefore, it is generalized that **T** is attained by inversing (I - R): $\mathbf{T} = (\mathbf{I} - \mathbf{R})^{-1}$.

VITA SHEET

Title of thesis	PLANNING AND OPTIMIZATION FOR LOGISTICS MANAGEMENT IN THE FOOD INDUSTRY
Major professor	John P. Norback
Major	Food Science
Minor	Distributed
Name	Hong K. Chung
Place and date of birth	Seoul, Rep. of Korea, December 3, 1960
Colleges and universities: years attended and degrees	Seoul National University, Korea, 1979-83, B.S. Seoul National University, Korea, 1983-85, M.S. University of Wisconsin-Madison, 1986-91, Ph.D. 1989-90, M.B.A.
Memberships in learned or honorary societies	
Publications	A study on the effects of sodium nitrite on lipid oxidation of pork during cooking, J. the Korean Agr. Chem. Soc. 29(2):148-158(1986). A clustering and insertion heuristic applied to a large routing problem in food distribution, J. Opl Res. Soc. 42:101-107(1991).
Current date	April 5, 1991

F-5266